Permutations and Combinations

Let X be a finite set with $|X| = n$.

A <u>k-permutation</u> of X is an ordered arrangement of k distinct elements of X.

A <u>k-combination</u> of X is an unordered arrangement of k distinct elements of X.

$P(n,k)$ = number of k-permutations of X

$C(n,k) = \binom{n}{k}$ = number of k-combinations of X

<u>Observe</u>: $C(n,k)$ is the number of k-element subsets of X.

<u>Theorem 1</u> For $k \leq n$

(a) $P(n,k) = \dfrac{n!}{(n-k)!}$ (b) $C(n,k) = \binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

<u>Theorem 2</u> For $0 < k < n$, $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

And $\binom{n}{k}$ is the number in position (n,k) of Pascal's triangle.

<u>Example</u> If m and n are non-negative integers then, in the integer grid, the number of sg-paths from $(0,0)$ to (m,n) is $\binom{n+m}{m} = \dfrac{(n+m)!}{n!\,m!}$.