

## Inverse Functions

If  $f: A \rightarrow B$  and  $g: B \rightarrow A$   
 $\uparrow$                        $\uparrow$   
domain(f)          codomain(f)

are functions which satisfy

$$g(f(a)) = a \quad \forall a \in A$$

and

$$f(g(b)) = b \quad \forall b \in B$$

Then  $g$  is called the inverse function of  $f$ .

(write  $g = f^{-1}$ )

(Also note that  $f$  is the inverse function of  $g$ .)

When does a function  $f: A \rightarrow B$  have an inverse function? answer: when  $f$  is a bijection.

example  $W_n = \{\text{bit strings of length } n\}$

$$|W_n| = 2^n$$

Define  $F: W_n \rightarrow W_n$  by taking  $F(w)$  to be the string obtain from  $w$  by replacing 0's with 1's and 1's with 0's.

$$F(011011) = 100100 \in W_6$$

$$F(1111111) = 0000000 \in W_7$$

Observe that

$$F(F(w)) = w \quad \forall w \in W_n$$

So  $F: W_n \rightarrow W_n$  is its own inverse.

that is  $F^{-1} = F$ .