Injective/Surjective 2

In practice how do we work with the definitions of injective and surjective?

To show $f: X \to Y$ is not injective

For this you just need to find two specific elements x_1 and x_2 with $x_1 \neq x_2$ for which $f(x_1) \neq f(x_2)$.

(ie. — give a "counter-example".)

To show $f: X \to Y$ is injective

For this you need to prove one of the implications

$$\big(f(x_1) = f(x_2)\big) \Rightarrow (x_1 = x_2) \quad , \quad \text{or},$$

$$(x_1 \neq x_2) \Rightarrow (f(x_1) \neq f(x_2))$$

is true.

(note: These two implications are contrapositives of each other.)

To show $f: X \to Y$ is not surjective

For this you need to find an element $y \in Y$ which does not equal $f(x)$ for any $x \in X$.

(There are two parts to this. First find the candidate element $y \in Y$. Then prove it doesn't equal $f(x)$ for any x.)

To show $f: X \to Y$ is surjective

For this you need to prove that for any element $y \in Y$ there is an element $x \in X$ with $f(x) = y$.