Consider functions $f: X \to Y$ where X and Y are finite sets with $|X| = m$ and $|Y| = n$.

- The number of functions from X to Y is n^m.
- The number of injective functions $f: X \to Y$ is denoted by $P(n,m)$ and

$$P(n,m) = \begin{cases} n!/(n-m)! & \text{if } m \leq n \\ 0 & \text{if } m > n \end{cases} \quad \circledast$$

$\circledast$ The fact that $P(n,m) = 0$ for $m > n$ is a version of the 'pigeon-hole principle'.

- The number of bijections $f: X \to Y$ equals $n!$ if $m = n$ and 0 if $m \neq n$. $(**)$
- The number of surjective functions $f: X \to Y$ is

$$\begin{cases} 0 & \text{if } m < n \\ n! & \text{if } m = n \\ \text{complicated} & \text{if } m > n \end{cases}$$

$(**)$ Important observation: A function $f: X \to Y$ between finite sets X and Y is injective iff it is surjective.