Problem How many different words can be formed using all 11 letters of "MISSISSIPPI"?

There are four letters

M — 1
I — 4
S — 4
P — 2

answer: 34,650

Some examples would be:

MIIIISSSSPP, MISPISISPIS, PPSSSSIIIIM, ...

Each of these can be viewed as an 11-permutation with repetition of the 4-element set $X = \{M, I, S, P\}$.

There are $4^{11} = 4{,}194{,}304$ such 11-permutations but most of them will not be solutions to the problem at hand (MMMMMMMMMMM, for example).

Approach 1: _ _ _ _ _ _ _ _ _ _ _

step1:

Choose 1 of 11 slots in which to place M $\binom{11}{1}$

step2

Choose 4 of 10 remaining slots for I's $\binom{10}{4}$

step3

Choose 4 of 6 remaining slots for S's $\binom{6}{4}$

step4

Choose 2 of 2 remaining slots for P's $\binom{2}{2} = 1$

$$\text{answer} = \binom{11}{1}\binom{10}{4}\binom{6}{4}\binom{2}{2} \qquad 0! = 1$$

$$= \frac{11!}{1!\,\cancel{10!}} \cdot \frac{\cancel{10!}}{4!\,\cancel{6!}} \cdot \frac{\cancel{6!}}{4!\,\cancel{2!}} \cdot \frac{\cancel{2!}}{2!\,\cancel{0!}}$$

$$= \frac{11!}{1!\,4!\,4!\,2!} = 34{,}650$$

multinomial coefficient

$1+4+4+2 = 11$

Approach 2:

Consider the set

$$Y = \{M_1, I_1, I_2, I_3, I_4, S_1, S_2, S_3, S_4, P_1, P_2\}$$

$|Y| = 11$. Consider 11-permutations in Y (w/o repetition). There are $11! = 39{,}916{,}800$

$M_1 P_1 S_3 S_4 I_2 I_1 S_2 I_3 I_4 S_1 P_2$

Now remove subscripts to get a solution to original problem

_ _ _ _ I_2 I_1 _ I_3 I_4 _ _

$I_2 I_4$ $\quad$ $I_3 I_1$

There are 4! to rearrange subscripted all of which determine the same solution to original problem

$$\frac{11!}{4!\,4!\,2!\,1!} = 34{,}650$$