Some Guidelines for Constructing and Writing Proofs

1. A proof really just consists of a sequence of sentences each of which follows from the preceding ones by common-sense logic incorporating references to definitions, hypotheses or previously proven theorems.

2. Before starting to organize the ideas for your proof make sure that you clearly understand the hypothesis (or hypotheses) and the conclusion of the statement to be proven. Determine all of the key definitions that play a role in the statement to be proven, and make sure you are familiar with these definitions before starting to construct your proof. Just thinking about the key definitions should start to give you ideas about how to construct your proof.

3. Do scratch work on the side to think out the line of explanation that you will use. Do this before starting to write your final proof. For more difficult proofs it is often necessary to do two or more rounds of scratch work before writing the final version. Your scratch work may incorporate Venn diagrams or other schematic diagrams, whatever helps you to organize the ideas.

4. In organizing your proof it’s almost always good to start by examining the desired conclusion and thinking about what will need to be done to verify it. In other words, in your scratch work start at the end and work backwards.

5. Are there any previously proved theorems that you might use to shorten your proof?

6. Always keep the intended reader in mind when you are writing and try to make it easy for them to follow what you are doing. This can be difficult, but try to view your final written proof from the perspective of a reader who has not spent as much time thinking about the problem as you have.

7. Make sure that your final written proof starts with a complete statement of the result that you intend to prove. This is important so that both the reader and you will have a clear statement of the desired outcome.

8. If you are using unusual definitions are worry that the reader may not know or remember a key definition then provide a formal statement of the definition.

9. In your final proof, be sure to write in complete and grammatically correct sentences. Remember that a sentence starts with a capital letter and ends with a period. It has a verb, a subject and a predicate.

10. Use the first sentence or two of your proof to declare any variables that you will use. If there is some object that you will refer to frequently throughout the proof then you should consider introducing a variable for that object so that you can refer to it easily during the proof. After declaring variables, use the next sentence or two to write out the hypotheses of the statement you are proving.
11. A ‘direct proof’ is a proof in which you start by assuming the hypothesis or hypotheses to be true and show that the desired conclusion can be reached by applying a sequence of logic explanations. Other methods of proof are very convenient in certain circumstances, such as ‘proof by contradiction’, ‘proof by contraposition’ and ‘mathematical induction’. If you decide to use one of these non-direct proofs then clearly indicate at the beginning which method you intend to use.

12. It is often nice to devote the last sentence of your proof to summarizing or partly recapping your line of argument. And then end with ‘□’ or ‘QED’.

13. Try to avoid overuse of mathematical or logical symbols in your writing. Be particularly cautious about starting a sentence with a symbol, or about allowing a sentence’s verb to be encoded in a symbol. Notice that different symbols play different grammatical roles in a sentence. Some are verbs, such as ‘=’ (the equal sign), ‘<’ (the less than symbol) ‘⊆’ (the subset symbol) or ‘∈’ (the element symbol). Others are nouns, such as ‘+’ (addition), ‘∩’ (set intersection) or ‘∪’ (the union symbol). (For example ‘a + b’ means ‘the sum of a and b’).

14. When your proof has different logical parts or cases then break it accordingly into separate paragraphs. For instance, if a theorem states that two sets are equal you might break the proof into four separate paragraphs as follows: (1) define variables and state hypotheses (if any), (2) show that the first set is a subset of the second, (3) show that the second set is a subset of the first, and (4) a short final recap where you explain that the two sets are equal since each is a subset of the other.

15. When you have completed your proof it is a good idea to look back and check where all of the key definitions were used. Also check to see if all of the hypotheses were used.