Discrete Math Group Project #8
10/16/20

Instructions: Reports will be due by 6PM on Wednesday, 10/21. Make sure to include a title at the top of your report with the names of all participating team members. If you submit via email, please title your file as 8 "Project8-Team*.pdf" (where * indicates your team number).

PART I: Follow-up to Exam 1

PROBLEM #1. Let B and C be sets. Problem 4 on the exam involved the proposition "*If* $B \subseteq C$ *then* $B - C = \emptyset$."
(a) Write a formal proof showing that the proposition is true.
(b) Write a formal proof showing that the converse of the proposition is also true. That is, "*If* $B - C = \emptyset$ *then* $B \subseteq C$."

PROBLEM #2. Work problem 6 from the exam.
We say that an sg-path in the integer grid has property G provided that there are no two successive U's in the RU-string associated with the path.
(a) List the RU-strings for all of the sg-paths from $(0,0)$ to $(2,2)$ that have property G.
(b) There are 120 sg-paths from $(0,0)$ to $(3,7)$. How many of them have property G? Explain.
(c) There are 120 sg-paths from $(0,0)$ to $(7,3)$. How many of them pass through the grid point $(2,2)$ and have property G? Explain.
(d) How many of the sg-paths from $(0,0)$ to $(7,3)$ have property G?

PROBLEM #3. Briefly discuss and compare notes with your teammates on the other problems from the exam. Write one or two comments about this.

PART II: Complete as much of this as you can.
In this part we will denote the set of all sg-paths from $(0,0)$ to a point (m,n) in the first quadrant (where $m \geq 0$ and $n \geq 0$) $SG(m,n)$. Following our usual convention we will denote each element of $SG(m,n)$ by a string σ of R's and U's. This string σ will have length $m+n$, the number of R's will equal m, and the number of U's will be n.
(a) Let $F_{(m,n)} : SG(m,n) \to SG(m',n')$ be the function which associates with each RU-string in $SG(m,n)$ the string which is obtained by replacing each R with R and each U with UR. What are the values of m' and n'?
(b) For this part let $(m,n) = (2,3)$. Show that there is a string σ with $F_{(2,3)}(\sigma) = RURRURUR$. Explain why there is no string σ in $SG(2,3)$ with $F_{(2,3)}(\sigma)$ equaling either $RRURUURR$ or $RURURRRU$.
(c) What conditions on a string in $SG(m',n')$ will guarantee that it does equal $F_{(m,n)}(\sigma)$ for some $\sigma \in S(m,n)$?
(d) Let $G_{(m,n)}(\sigma)$ be the RU-string obtained by removing the last letter of the string $F_{(m,n)}(\sigma)$. In the case where $(m,n) = (2,3)$ describe which RU-strings will equal $G_{(m,n)}(\sigma)$ for some $\sigma \in SG(m,n)$?
(e) Can you see a connection between this problem and part (d) of problem 2?