Discrete Math Group Project #5
9/25/20

Instructions: Reports will be due by Wednesday 9/30. Make sure to include a title at the top of your report with the names of all participating team members. If you submit via email, please title your file as "Project4-Team*.pdf" (where * indicates your team number).

PART I:

PROBLEM #1. At the start of the semester Professor A tells her class:

"*You will pass this course if you score* 70 *or higher on the final exam.*"

(a) Professor A's statement has the form "if $\mathcal{P}_A$ then $\mathcal{Q}_A$". What are the statements $\mathcal{P}_A$ and $\mathcal{Q}_A$?
(b) A student scores 91 on the final and gets a "C" in the course. Does this violate Professor A's statement? (In other words, is Professor A's statement false in this case?)
(c) A student scores 61 on the final and gets a "C". Does this violate Professor A's statement?
(d) A student scores 63 on the final and gets an "F". Does this violate Professor A's statement?
(e) After the semester a student determines that Professor A lied to them, what does that tell you about that student's final and course grades?
(f) Write a concise sentence that expresses the statement "$\mathcal{Q}_A$ implies $\mathcal{P}_A$", and then answer the question in part (e) if the professor had made this statement at the start.

PROBLEM #2. At the start of the semester Professor B tells her class:

"*You won't pass this course unless your final exam score is at least 70.*"

(a) Professor B's statement has the form "if $\mathcal{P}_B$ then $\mathcal{Q}_B$". What might the statements $\mathcal{P}_B$ and $\mathcal{Q}_B$ be? How do these two statements relate to $\mathcal{P}_A$ and $\mathcal{Q}_A$?
(b) After the semester a student determines that Professor B lied to them, what does that tell you about that student's final and course grades?
(c) Are Professor A's statement and Professor B's statement the same? How does Professor B's statement compare to the statement in part (f) of the previous problem.

PART II:

PROBLEM #3. Let p and q be statements. One of DeMorgan's laws in logic says that the compound statement "$\neg(p \vee q) \iff ((\neg p) \wedge (\neg q))$" is a tautology. Use a truth table to verify this.[1] Then write as concise a sentence as you can that describes what the corresponding law of inference would say.

PROBLEM #4. Work the following problems from Hammack's book:[2]
section 2.5, page 50: # 10, 11
section 2.6, page 52: # 12, 13, 14

PART III:

Let n and m be integers with $n > 0$. We say that m is divisible by n provided that the remainder when m is divided by n equals 0. (Equivalently, this means that there is an integer q such that $m = qn$.)

PROBLEM #5. Explain why the sum of three consecutive integers is always divisible by 3.

PROBLEM #6. Either prove or disprove the statement: *For each integer $k > 0$ the sum of k consecutive integers is divisible by k.*

PROBLEM #7. Can you characterize all values of k for which the sum of k consecutive integers is divisible by k? Justify your answer.

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[1] The logical symbol "$\neg$" denotes negation but some authors (such as Hammack) use "$\sim$" in its place. It is also commonplace to use "$\rightarrow$" and "$\leftrightarrow$" for "$\implies$" and "$\iff$".

[2] To say that two statements p and q are logically equivalent (which Hammack expresses as $p = q$) is the same as saying that $p \iff q$ is a tautology.