Discrete Math Group Project #4
9/18/20

Instructions: Reports will be due by Wednesday 9/23, and may be submitted either electronically by 6pm, or in written form at class. If you submit via email, please title your file as "Project4-Team*.pdf" (where * indicates your team number). Make sure you have a title at the top of your report which includes the names of all participating team members.

PART I: Consider these twenty sentences:

1. -3 is an odd integer or 14 is an odd integer.
2. -3 is an odd integer or 15 is an odd integer.
3. It is raining outside.
4. t^2 is a real number.
5. If t is a real number then t^2 is a real number.
6. If x is an integer then $(-x^2 - x + 6)/(x - 2))$ is an integer.
7. What is discrete mathematics about?
8. Discrete mathematics is fun.
9. If $2 + 5 = 12$ then $4 + 8 = 23$.
10. If p is a natural number which is prime then $4p^2 + 1$ is natural number which is prime.
11. Every integer is a rational number.
12. $\sqrt{300}/\sqrt{75}$ is a positive integer.
13. It is not true that every real number x satisfying $(x - 1)(x^2 + x - 1) = 0$ is an integer.
14. Each even integer larger than 2 can be expressed as the sum of two prime integers.
15. An integer is divisible by 12 only if it is divisible by 6.
16. This sentence is false.
17. Let f be a function of a real variable (like in calculus 1). If x and t are real numbers with $f(x) = f(t)$ then $x = t$.
18. Let f be a function of a real variable. If x and t are real numbers with $x = t$ then $f(x) = f(t)$.
19. 12 is even or 8 is odd, and -13 is even.
20. 12 is even, or 8 is odd and -13 is even.

Problem #1. For each of the sentences that are statements, which are True and which are False?

Problem #2. Two of the sentences are not statements. Which are these?

Problem #3. Which if any of the answers to Problem #1 do you have disagreements about within your group? Any comments?

PART II:

PROBLEM #4. Consider problems 11-14 on page 24 of Hammack's book.

(a) Solve the four problems.

(b)* Find as many as you can of the four sets that can be described using only union "$\cup$" and set difference "$-$".

(c)* Find as many as you can of the four sets that can be described using only intersection "$\cap$" and set difference "$-$".

(d)* Find as many as you can of the four sets that can be described using only intersection "$\cap$" and union "$\cup$".

(*These problems may be hard to solve definitively, but see what you can come up with.)

PART III:

Let n and $m > 0$ be integers. The division algorithm says that there are unique integers r and q where $0 \leq r < m$ and $n = qm + r$. Here the non-negative integer r is referred to as the remainder of n when divided by m. (For example, the remainder when 81 is divided by 6 equals 3 because $81 = 6 \cdot 13 + 3$.)

PROBLEM #5. (a) List all of the integers n between -20 and 20 inclusive that have a remainder of 3 when divided by 7.

(b) Let S be the set of all integers n which have a remainder of 3 when divided by 7. How many elements does S have? Give a concise description of S using set builder notation.

PROBLEM #6. (a) Give a convincing explanation that the following is false: *For any integers a and b, if both a and b have a remainder of 2 when divided by 5 then ab also has a remainder of 2 when divided by 5.*

(b) Given that a and b are integers which both have a remainder of 2 when divided by 5 find a true statement that does describe the remainder when ab is divided by 5. Write a few sentences that justify your claim.