Discrete Math Group Project #3
9/11/20

Instructions: Be sure to complete all four parts of this project. Reports will be due by Wednesday 9/16, and may be submitted either electronically by 6pm, or in written form at class. If you submit via email, please title your file as "Project2-Team*.pdf" (where * indicates your team number). Make sure you have a title at the top of your report which includes the names of all participating team members.

Intro: Take the first few minutes to get acquainted with classmates in your team.

Part I: Consider the following statement and its "proof".

CLAIM 1. *There is no integer larger than* 1.

Proof. Let n be the largest positive integer. Since every positive integer is greater or equal than 1, we know that $n \geq 1$. Multiplying both sides of this inequality by the positive integer n shows that $n^2 \geq n$. However we also know that $n \geq n^2$ because by assumption n is the largest positive integer. From these two inequalities we conclude that n^2 and n must be equal. Now dividing both sides of the equation $n^2 = n$ by n (which is positive and therefore not equal to 0) we obtain that $n = 1$. So 1 is the largest integer which means that there is no integer larger than 1. □

PROBLEM #1. (a) Obviously this claim is not true so there must be a mistake in its purported "proof". Locate the mistake and explain why the proof doesn't work.
(b) Even though the "proof" is invalid, each sentence logically implies the succeeding sentence. Do you agree?

Part II:

PROBLEM #2. Give answers for all of the problems in Problem 3 of the "Exercises for Section 1.5" on page 19 of Hammack's book. Just listing answers will be OK for this problem (that is, if you get them correct).

Part III:

PROBLEM #3. (a) In class we briefly discussed the set $\{7a + 3b \mid a, b \in \mathbb{Z}_{\geq 0}\}$. Describe the non-negative integers which are not in this set. Describe the non-negative integers which are in this set. Write three or four sentences outlining how to convince someone that your answers are correct.
(b) List the non-negative integers which are not in the set $\{9a + 10b + 11c \mid a, b, c \in \mathbb{Z}_{\geq 0}\}$.

Part IV:

Let A and B be sets. A **function f from A to B** is a rule that assigns to each element of A an element of B. If $a \in A$ then the element of B to which it is assigned is denoted by $f(a)$. We write $f : A \to B$ to signify that f is a function from A to B, and refer to A as the **domain of f** and B as the **co-domain of f**.

A function $f : A \to B$ is a **relabeling**[1] if it has the property that every element $b \in B$ equals $f(a)$ for one and one element $a \in A$.

PROBLEM #4. Let $\mathcal{P}$ be the set of all *sg* paths in the integer grid starting at $(0, 0)$ and ending at a point in the first quadrant, and let $\mathcal{S}$ be the set of all finite strings of R's and U's.

(a) Let $f : \mathcal{P} \to \mathcal{S}$ be the function that assigns an RU-string to each sg-path in $\mathcal{P}$, as previously described in project #1. Is f a relabeling? Explain briefly.

(b) Let $g : \mathcal{S} \to \mathcal{S}$ be defined by replacing each R in an RU-string with U and each U with R. Is g a relabeling? If p is an sg-path from $(0, 0)$ to (a, b) associated with the RU-string s, what can you say about the path associated with $g(s)$?

(c) Let $h : \mathcal{S} \to \mathcal{S}$ be the function defined by replacing each R in an RU-string with RR and each U with U. Is this a relabeling? If p is an sg-path from $(0, 0)$ to (a, b) associated with the RU-string s what can you say about the path associated with $g(s)$? Give some examples to illustrate.

(d) Each sg-path p in $\mathcal{P}$ has an associated ending grid point (a, b). Describe this as a function between two sets (and clearly indicate the domain and co-domain sets). Is this function a relabeling? Explain.

[1] This term was used in class to indicate when two sets have the same cardinality: to say that sets A and B have the same cardinality means that there is a relabeling $f : A \to B$. A relabeling is also referred to as a "one-to-one correspondence".