Discrete Math Group Project #1 - 8/28/20

Instructions: Each team should submit one solution sheet for this assignment, due by class time on Monday, 8/31. The solution may be submitted either electronically in pdf form, or in written form at Monday's class. If you submit via email please title your file as "Project1-Team*" (where * indicates your team).

Part I:

Take the first 10 minutes or so to introduce yourself, and get acquainted with your group mates. Take time to discuss things like: why you are taking this course and how it fits into your academic plans; how long you've been at OU; where you're from; or etc.

Part II:

Consider the following implication statements in which the variable x denotes a real number:

(i) If $x = 3$ then $x^2 = 9$.

(ii) If $x^2 = 9$ then $x = 3$.

(iii) If $x \neq 3$ then $x^2 \neq 9$.

(iv) If $x^2 \neq 9$ then $x \neq 3$.

(a) Write down both the hypothesis and the conclusion of statement (i).

(b) Write down both the hypothesis and the conclusion of statement (ii).

(c) Write down both the hypothesis and the conclusion of statement (iii).

(d) Write down both the hypothesis and the conclusion of statement (iv).

(e) Which of the four statements are mathematically correct? For one of the incorrect statements write a sentence or two to convince someone that it is false.

Part III:

The problems in this part involve the integer grid consisting of all vertical lines in the xy-plane with equation $x = a$ where $a \in \mathbb{Z}$, together with all horizontal lines with equation $y = b$ where $b \in \mathbb{Z}$. The points (a, b) where both a and b are integers will be called 'grid points'. A 'grid path' between two grid points consists of a string of horizontal or vertical line segments of length one where the end point of one line segment is the starting point of the next one, and the number of edges in the string is the 'length' of the grid path. We will restrict attention to grid paths which start at the origin $(0, 0)$ and end at a point (a, b) in the first quadrant, and if this grid path has shortest possible length we will call it an 'sg-path'. (The first quadrant is assumed to include the positive x- and y-axes.) The basic problem is to determine how many sg-paths there are from the origin to a given grid point (a, b) (in the first quadrant).

In your work, you may find it very convenient to identify each sg-path of length n which starts at the origin with a string of n letters consisting of R's and U's. Here R denotes "move one step to the right" and U denotes "move one step up".[1] For example, the string $RRURUUUU$ would describe the sg-path of length 8 which starts at $(0, 0)$, moves 2 steps to the right, 1 step up, 1 step to the right, and then 4 steps up, ending at the point $(3, 5)$. As another example, there are six sg-paths from the origin to $(2, 2)$: $RRUU$, $RURU$, $RUUR$, $URRU$, $URUR$ and $UURR$.

(a) How many sg-paths are there from the origin to $(4, 2)$? After you determine the answer, write a few sentences that could serve to convince someone that your answer is correct.

(b) How many sg-paths are there from the origin to $(2, 4)$? In comparison with (a), does this suggest a general principle?

(c) If an sg-path starts at the origin and has length 6, what are its possible endpoints (a, b)? For each of these different possible endpoints (a, b), how many sg-paths are there that go from the origin to (a, b).

(d) Make a list of the numbers that you discovered in (c). What is the sum of the numbers in your list? You should see a connection between this list of numbers and the first few rows of Pascal's triangle. What is it?

(e) Repeat questions (c) and (d) for sg-paths of length 4 starting at $(0, 0)$ and see where that occurs in Pascal's triangle.

(f) Use Pascal's triangle to conjecture how many sg-paths there are from the origin to $(5, 4)$.

(g) Find at least three points (a, b) in the first quadrant for which the number of sg-paths from the origin equals 126. Can you conjecture how many such points (a, b) there are in total?

[1] An sg-path won't have any steps to the left, or any steps down, because it is required to be a shortest path from the origin to (a, b).