Math 2513
Homework Assignment #1
for class June 7 (but not to turn in)

For each nonnegative integer, the Catalan number C_n is defined to be the number of allowable staircases in the n-by-n square S_n. (A staircase is a path connecting the upper left corner L to the lower right corner R in S_n. A staircase is allowable if (1) it always moves downward or to the right as it progresses from L to R, and (2) it never crosses over the diagonal from L to R.) In class we discussed the recursion formula

$$C_n = C_0C_{n-1} + C_1C_{n-2} + C_2C_{n-3} + \cdots + C_{n-1}C_0$$

which can be used to determine these numbers. We also showed that $C_0 = C_1 = 1$, $C_2 = 2$ and $C_3 = 5$.

Problems:
(1) Draw the 14 allowable staircases in S_4.
(2) Use the recursion formula to determine C_{10}.
(3) How many allowable staircases in S_{10} hit the diagonal at the point which 4 squares down and 4 squares to the right of L?
(4) What is the smallest value of n for which C_n exceeds 1 million?