EXAM 1 Name ______________________________
Math 2513
6-17-05

1. *(10 points)* Consider the three sets $\emptyset$, $\{\emptyset\}$ and $\{\{\emptyset, \{\emptyset\}\}\}$.
   (a) Briefly explain why no two of these three sets are equal.
   (b) Determine the cardinality of each of the three sets.

2. *(15 points)* Use a proof by contradiction to show that $A \cap \overline{A} = \emptyset$ for any set A.

3. *(15 points)* Let p and q be propositions.
   (a) Construct a truth table for the compound proposition $(\neg q \to p) \wedge (q \vee \neg p)$.
   (b) Is the proposition $(\neg q \to p) \wedge (q \vee \neg p)$ logically equivalent to $p \to q$? Refer to (a) to justify your answer.

4. *(15 points)* Prove the statement: *For all sets A, B and C, if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.*

5. *(10 points)* Show that the following statement is false:
   For all sets A, B and C, if $A \subseteq B$ and $A \subseteq C$ then $B \subseteq C$.

6. *(20 points)* Let $f : \mathbb{R} \to [0, \infty)$ be the function described by the rule $f(x) = |x| + 1$.
   (a) Identify the domain and the codomain of f.
   (b) Determine the range of f.
   (c) Is f onto? Justify your answer using the definition of onto.
   (d) Is f one–to–one? Justify your answer using the definition of one–to–one.

7. *(15 points)* (a) State what it means for two sets C and D to be equal.
   (b) Prove that $A = (A \cap B) \cup (A - B)$ for all sets A and B.