Exam 2
Solutions to first two problems
Math 2513

1. Show that the composition of two one–to–one functions is one–to–one.

Proof. Let A, B and C be sets, and let $f : A \to B$ and $g : B \to C$ be functions. Assume that f and g are one–to–one functions. We will show that $g \circ f : A \to C$ is one–to–one. Suppose that a_1 and a_2 are elements of A for which $g \circ f(a_1) = g \circ f(a_2)$. Then using the definition of composition of functions

$$g(f(a_1)) = g \circ f(a_1) = g \circ f(a_2) = g(f(a_2)).$$

Since $g : B \to C$ is a one–to–one function and $f(a_1)$ and $f(a_2)$ are elements of B which get sent by g to the same element of C, it follows that $f(a_1) = f(a_2)$ (using the definition of one–to–one). Since $f : A \to B$ is a one–to–one function and a_1 and a_2 are elements of A which get sent by f to the same element of B, it follows that $a_1 = a_2$ (using the definition of one–to–one). Therefore we have shown that: if $g \circ f(a_1) = g \circ f(a_2)$ then $a_1 = a_2$. By the definition of one–to–one it follows that $g \circ f$ is one–to–one. □

COMMENTS:
i) Always remember that the definition of one–to–one is an if/then statement.
ii) The goal in this proof is to show that $g \circ f$ is one–to–one, which means that "if $g \circ f(a_1) = g \circ f(a_2)$ then $a_1 = a_2$". Therefore, once you have declared variables and stated the hypotheses at the beginning of the proof, the next thing to do is to suppose that a_1 and a_2 are elements of A with $g \circ f(a_1) = g \circ f(a_2)$. Then, to complete the proof, you will need to show that a_1 and a_2 are equal.
iii) I chose to use letters A, B, C, f, g, a_1 and a_2 in my proof but any other variables could have used in their place since the original problem didn't set any variable names.
iv) Note that the definition of one–to–one was invoked three times in the proof.

2. For each of the following statements, write an English statement which describes the negation of that statement in the most direct way.
a) This week it will rain on Sunday.
b) This week it will rain on Saturday and Sunday.
c) If it rains on Saturday this week then it will not rain on Sunday.

ANSWERS:
a) This week it will not rain on Sunday.
b) This week it will not rain on at least one of Saturday or Sunday.
c) This week it will rain on both Saturday and on Sunday.