Class Problem
Math 2513
Wednesday, July 6

PROBLEM. Let A be a set with n elements which are labelled $a_1, a_2, \ldots, a_n$. If B is a subset of A let $f(B)$ be the bit string of length n which has a 1 in the ith position if $a_i \in B$ and has a 0 in the ith position if $a_i \notin B$. This defines a function f from the power set of A to the set $\mathcal{B}_n$ consisting of all bit strings of length n. That is $f : \mathcal{P} \to \mathcal{B}_n$.
(a) In the case where $n = 8$, determine each of the following:

$$f(\emptyset),\ f(A),\ f(\{a_5\}),\ f(\{a_8\}) \text{ and } f(\{a_1, a_3, a_8\}).$$

(b) In the case where $n = 8$, describe the subsets B for which $f(B)$ is each of:

$$10101010,\ 01010101,\ 11110000,\ \text{ and } 00001111.$$

(c) How can the cardinality of a subset B be determined by examining its corresponding bit string $f(B)$?

ANSWERS:

(a) $f(\emptyset) = 00000000$, $f(A) = 11111111$, $f(\{a_5\}) = 00001000$, $f(\{a_8\}) = 00000001$ and $f(\{a_1, a_3, a_8\} = 10100001$.
(b) $f(\{a_1, a_3, a_5, a_7\}) = 10101010$, $f(\{a_2, a_4, a_6, a_8\} = 01010101$, $f(\{a_1, a_2, a_3, a_4\} = 11110000$, and $f(\{a_5, a_6, a_7, a_8\} = 00001111$.
(c) The cardinality of a subset B of A equals the number of 1's in the bit string $f(B)$.