Class Problem
Math 2513
Norman, July 14

**Problem.**
(a) How many "words" of length 14 can be made using the letters of NORMANOKLAHOMA?
(b) Determine the number of 14-letter words formed from NORMANOKLAHOMA contain three consecutive O’s.
(c) How many "words" of length 14 made from NORMANOKLAHOMA have no consecutive O’s?

**Solutions:**
(a) By the Multinomials Theorem, the number of permutation of \{N, O, R, M, A, K, L, H\} with three O’s and A’s, two N’s and M’s and one R, K, L and H is
\[
\binom{14}{3, 3, 2, 2, 1, 1, 1, 1} = \frac{14!}{3!3!2!2!} = 605,404,800.
\]

**Note:** The numerical value is only included to give a relative idea of the size of the number.

(b) The number of permutation of \{N, OOO, R, M, A, K, L, H\} with three A’s, two N’s and M’s and one OOO, R, K, L and H is
\[
\binom{12}{3, 2, 2, 1, 1, 1, 1} = \frac{12!}{3!2!2!} = 19,958,400.
\]

(c) We break the task of listing words of the indicated type with no consecutive O’s into two subtasks:
First choose an 11-letter word with three A’s, two N’s and M’s and one R, K, L and H. Second, choose 3 of the 12 gaps in the word resulting from the first task and insert one O into each of these 3 gaps. There are \(\binom{11}{3,2,2,1,1,1,1}\) to perform the first task and \(\binom{12}{3}\) ways to perform the second. So the answer to (c) is
\[
\binom{11}{3, 2, 2, 1, 1, 1, 1} \binom{12}{3} = \frac{11!}{3!2!2!} \cdot \frac{12!}{3!2!2!3!} = 365,904,000.
\]