Class Problem
Math 2513
Monday, July 11

PROBLEM. How many permutations of the letters $ABCDEFGH$ contain:
(a) the string CDE?
(b) the strings BA and FGH?
(c) at least one of the strings BA or FGH?
(d) the strings FGH and DG?
(On one of these four problems you should have used the Principle of Inclusion/Exclusion. Be sure to indicate where it was used.)

NOTE: Since there are 8 letters available, the total number of possible permutations of the letters is $8! = 40,320$. Are your answers smaller than this?

ANSWERS:

(a) The set of permutations of the letters $ABCDEFGH$ that contain the string CDE is the same as the set of permutations of the 6-element set $\{A, B, CDE, F, G, H\}$. The latter set has $P(6,6) = 6! = 720$ elements.

(b) Here we are interested in the set of permutations of the set $\{BA, C, D, E, FGH\}$, of which there are $5! = 120$.

(c) Let S be the set of permutations of $ABCDEFGH$ that contain BA and let T be the set of permutations of $ABCDEFGH$ that contain FGH. Then (as above) $|S| = 7!$ and $|T| = 6!$ and $|S \cap T| = 5!$. Using the principle of inclusion/exclusion, the cardinality of $S \cup T$ equals

$$|S \cup T| = |S| + |T| - |S \cap T| = 7! + 6! - 5! = 5,640.$$

(d) There are no permutations of $ABCDEFGH$ that contain both strings FGH and DG, because the letter G can't be in two places at once.