Class Problem
Math 2513
Thursday, June 9

PROBLEM. Let A and B be sets. Prove that $A \cup (A \cap B) = A$.

Solution:

Theorem: Let A and B be sets, then $A \cup (A \cap B) = A$.

Proof. Let A and B be sets. Suppose that x is an element of $A \cup (A \cap B)$. From the definition of union it follows that (1) $x \in A$ or (2) $x \in A \cap B$. If (2) holds then x is an element of both A and B by the definition of intersection, and so $x \in A$. In either case (1) or (2) we may conclude that $x \in A$. Therefore we have shown that every element of $A \cup (A \cap B)$ is also an element of A, and so $A \cup (A \cap B)$ is a subset of A (by the definition of subset).

Applying Theorem 1 (which was proved in class) to the sets A and $A \cap B$, we know that A is a subset of $A \cup (A \cap B)$. Since both $A \cup (A \cap B) \subseteq A$ and $A \subseteq A \cup (A \cap B)$, it follows by the definition of set equality that $A \cup (A \cap B) = A$, and this completes the proof. $\square$

For completeness here is a proof of Theorem 1:

Theorem 1: Let A and B be sets, then A is a subset of $A \cup B$.

Proof. Let A and B be sets. Suppose that t is an element of A. Then it is certainly true that t is an element of A or t is an element of B. Thus t is an element of $A \cup B$ by the definition of union. Since every element of A is also an element of $A \cup B$, we conclude that A is a subset of $A \cup B$, and the proof is finished. $\square$