Problem. Let $A = \{1, 2, 3, 4, 5\}$. The directed graph shown below determines a relation $R$ on $A$ using the convention that if there is a directed edge from $x$ to $y$ then $(x, y)$ is an element of $R$.

(a) What is $R$ for this example?
(b) Which of the following properties does this relation satisfy:

- reflexive, symmetric, anti-symmetric, transitive?

Justify each of your four answers with an explanation or a counterexample as appropriate.

\[ R = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4), (3,2), (3,3), (4,4), (5,4), (5,5)\} \]

(b) This relation does not satisfy any of the four properties.

1. $R$ is not reflexive because $(2, 2) \notin R$.
2. $R$ is not symmetric because $(1, 2) \in R$ but $(2, 1) \notin R$.
3. $R$ is not anti-symmetric because $(2, 3) \in R$ and $(3, 2) \in R$ but $2 \neq 3$.
4. $R$ is not transitive because $(3, 2) \in R$ and $(2, 4) \in R$ but $(3, 4) \notin R$.

NOTE: See if you can interpret the answers to part (b) in terms of the directed graph and in terms of the matrix for $R$, which is the $5 \times 5$ matrix:

\[
M = \begin{pmatrix}
1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]