Class Problem
Math 2513
Friday, June 24

Please feel free to collaborate and/or compare notes with your classmates on this problem.

PROBLEM. (a) Let $N$ be the positive integer $N = 25!$. Find all of the prime numbers which divide $N$, then work out the complete prime factorization of $N$.

(b) In part (a) you should have determined that the 7-part of 25! equals $7^3$ (see definition below). Are there any other positive integers $k$ besides $k = 25$ for which $7^3$ is the 7-part of $k$!? If so, find them.

Definition. Let $a$ be a positive integer whose prime factorization is

$$a = p_1^{n_1} p_2^{n_2} p_3^{n_3} \cdots p_r^{n_r}$$

where $p_1, p_2, \ldots, p_r$ are distinct primes and each $n_i$ is a natural number. Then the $p_i$-part of $a$ is defined to be $p_i^{n_i}$ for $i = 1, \ldots, r$.

ANSWERS:

(a) The prime divisors of $N = 25!$ are all of the prime numbers less than or equal to 25. These are 2, 3, 5, 7, 11, 13, 17, 19 and 23. The prime factorization of $N$ is

$$N = 2^{22} 3^{10} 5^6 7^3 11^2 13^1 17^1 19^1 23^1.$$  

Incidentally, 25! equals 15511210043330985984000000.

(b) If $k$ equals 21, 22, 23, 24, 25, 26 or 27 then the 7-part of $k$! is $7^3$. If $k$ is larger than 27 then the 7-part of $k$! is larger than $7^3$. (For example, the 7-part of 28! is $7^4$.) If $k$ is smaller than 21 then the 7-part of $k$! is less than $7^3$. (For example, the 7-part of 20! is $7^2$.)