Class Problem
Math 2513
Thursday, June 23

PROBLEM. Let a, b and c be integers.
(a) Show that if a divides b and b divides c then a divides c.
(b) Give an example showing that if a divides b and c divides b then a need not divide c.

SOLUTION:

(a) *Proof.* Let a, b and c be integers. Suppose that a divides b and b divides c. Since a divides b there is an integer k such that $b = a \cdot k$ (by the definition of "divides"). Likewise, since b divides c there is an integer ℓ such that $c = b \cdot \ell$. Putting these equations together we obtain

$$c = b \cdot \ell = (a \cdot k) \cdot \ell = a \cdot (k\ell).$$

Since $k\ell$ is an integer (the set $\mathbb{Z}$ of integers is closed under multiplication), this shows that a divides c. $\square$

(b) There are lots of possible answers. For instance, if $a = b = 2$ and $c = 1$ then a divides b (since $2 = 2 \cdot 1$) and c divides b (since $2 = 1 \cdot 2$) but a does not divide c (the only positive integer divisor of 1 is 1 itself since if n and m are positive integers and n divides m then $n \leq m$).