Class Problem
Math 2513
Tuesday, June 21

PROBLEM. Prove that if n is an integer and $3n+2$ is odd then n is odd. Use either an indirect proof or a proof by contradiction.

SOLUTION:

Indirect Proof. Let n be an integer. Assume that n is not odd. By definition of odd, this means that n is even. Therefore we may express n as $n = 2k$ for some integer k, and

$$3n + 2 = 3(2k) + 2 = 6k + 2 = 2(3k + 1).$$

Since $3k + 1$ is an integer it follows that $3n + 2$ is even (by the definition of even), and so $3n + 2$ is not odd. This completes the proof using the technique of indirect proof. (We have shown that if n is not odd then $3n + 2$ is not odd, which is the contrapositive of the statement to be proved.) $\square$

Proof by Contradiction. Let n be an integer. Suppose that $3n + 2$ is odd and that n is not odd. By definition of odd, this means that n is even. Therefore n equals $2k$ for some integer k (using the definition of even), and so

$$3n + 2 = 3(2k) + 2 = 6k + 2 = 2(3k + 1).$$

Since $3k + 1$ is an integer it follows that $3n + 2$ is even (by the definition of even). Therefore $3n + 2$, which is odd by our original hypothesis, is not odd, and this is a contradiction. This establishes the desired result that if $3n + 2$ is odd then n must be odd. $\square$