Class Problem
Math 2513 Monday, June 20

PROBLEM. Consider the statement: *Let A and B be sets. If A is a subset of $A - B$ then $A \cap B = \emptyset$.*
(a) Use a proof by contradiction to prove this statement.
(b) State the converse of this statement. Use Venn diagrams to guess whether the converse statement is true.

REMINDER: To prove an implication statement $p \to q$ there are three main approaches:
• Direct Proof: Assume p is true and then show that q is true.
• Indirect Proof: Assume q is false and then show that p is false.
• Proof by Contradiction: Assume p is true and q is false and then derive a contradiction.

SOLUTION:

(a)

Proof. Let A and B be sets. Assume that $A \subseteq A - B$ and that $A \cap B \neq \emptyset$. Since $A \cap B$ is nonempty it contains at least one element. Let $x \in A \cap B$ be such an element. By definition of intersection (1) $x \in A$ and (2) $x \in B$. Since $A \subseteq A - B$ and $x \in A$ (by (1)), the definition of subset implies that x must be an element of $A - B$. Therefore $x \in A$ and $x \notin B$ by the definition of set difference. It follows that $x \notin B$ which contradicts our previous conclusion (2) that $x \in B$. This completes the proof using the proof-by-contradiction method of proof. $\square$

(b) The converse statement is: *If $A \cap B$ equals the empty set then A is a subset of $A - B$.* This converse statement also turns out to be true.