

Class Problem
Math 2513
Friday, June 10

PROBLEM. Consider the statement:

Let A , B and C be sets. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

- (1) Notice that the statement is an implication statement. What are the hypotheses of this statement? What is the conclusion of this statement?
- (2) Prove the statement just using basic definitions from set theory.

Solution:

(1) The hypothesis of the statement is " $A \subseteq B$ and $B \subseteq C$ ". The conclusion is " $A \subseteq C$ ".

(2) **Theorem:** *Let A , B and C be sets. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.*

Proof. Let A , B and C be sets. Assume that A is a subset of B and that B is a subset of C . Suppose that x is an element of A . Using the fact that $A \subseteq B$ and the definition of subset, it follows that x is an element of B . Since $B \subseteq C$ and $x \in B$ then x is an element of C (this uses the definition of subset again). Therefore, it has been shown that each element of A is an element of C . From the definition of subset we conclude that A is a subset of C , and the proof is complete. \square