EXAM 3
Math 2513
11-19-04

1. *(15 points)*   Use a proof by contradiction to show that the sum of a rational number and an irrational number is irrational.

ANSWER: This problem was discussed in class.

2. *(10 points)*   Use the binomial theorem to determine the x^6y^3 term in the expansion of $(2x - 3y)^9$.

ANSWER: $C(9,3)(2x)^6(-3y)^3 = -145152x^6y^3$

3. *(20 points)*   Determine the number of bit strings which satisfy each of the following and clearly name any counting principles that are used:
   (a) The strings of length 8 which begin with 4 zeros and end with 101?
   (b) The strings of length 8 which begin with 4 zeros or end with 101?
   (c) The strings of length 8 which have exactly 4 zeros.

ANSWER: (a) 2,   (b) $2^4 + 2^5 - 2 = 46$ using inclusion/exclusion,   (c) $C(8,4) = 70$

4. *(15 points)*   (a) How many different strings of length 5 can be made from the letters in "BOBBY"
   (b) How many different strings of length 4 can be made from the letters in "BOBBY"

ANSWER: (a) 20,   (b) 20

5. *(20 points)*   Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4\}$.
   (a) How many functions from A to B are there?
   (b) How many 1-1 functions from A to B are there?
   (c) How many onto functions from A to B are there?
   (d) How many functions f from A to B are there which satisfy the condition that there are exactly two elements x in A with $f(x) = b_3$?

ANSWER: (a) $4^3 = 64$,   (b) $4 \cdot 3 \cdot 2 = 24$,   (c) 0,   (d) $C(3,2) \cdot 3 = 9$

6. *(20 points)*   Let R be the relation on $\mathbb{N}$ given by $R = \{(m, n) \mid m, n \in \mathbb{N}$ and $(m - n)(m - 1) = 0\}$. Explain what each of the following means, and then determine whether this relation R satisfies it:
   (a) R is reflexive.   (b) R is symmetric.   (c) R is anti-symmetric.   (d) R is transitive.

ANSWER: (a) YES,   (b) NO,   (c) YES,   (d) YES