EXAM 3 Retest Name ______________________

Math 2513

4-26-05

1. *(15 points)* (a) Carefully state the principle of inclusion–exclusion.
(b) Illustrate part (a) by counting the number of bit strings of length ten which contain at most one zero or which start and end with "111".

2. *(20 points)* Use the technique of mathematical induction to prove that $\sum_{j=1}^{n} 3j+1 = (3n^2+5n)/2$ for every integer $n \geq 1$.

3. *(10 points)* Determine the middle entry of the row of Pascal's triangle whose first two entries are 1 and 14. (NOTE: This row of Pascal's triangle has odd length, so there is a "middle" entry.)

4. *(15 points)* Let S be the set of all words of length 9 that can be formed using the letters a, e, k and y where repetition of letters is allowed.
(a) How many elements does S have?
(b) How many subsets with exactly 2 elements does S have? Show that there more than 10^9.
(c) Let S_1 be the subset of S which consists of those words where at most two of the four letters appear one of which is k. Determine $|S_1|$.
(d) How many elements of S contain a "*kayak*" subword?
(e) How many elements of S contain exactly four k's, two a's and one e?

5. *(10 points)* (a) How many bit strings contain exactly 10 zeros and 3 ones where every 1 is immediately preceded by two 0's? (b) How many bit strings contain exactly 9 zeros and 6 ones where every 1 is preceded by a 0 and each block of 0's has even length? List these bit strings.

6. *(10 points)* Prove the identity $C(2n,3) = 2C(n,3) + 2nC(n,2)$ where n is a positive integer.

7. *(10 points)* (a) Determine the coefficient of x^6y^4 in the expansion of $(x+y)^{10}$
(b) Determine the coefficient of x^6y^4 in the expansion of $(2x-y)^{10}$
(c) Determine the coefficient of x^1 in the expansion of $(x+\frac{1}{x^2})^{10}$

8. *(10 points)* How many solutions to the equation $n_1+n_2+n_3+n_4 = 15$ are there if
(a) n_1, n_2, n_3 and n_4 are nonegative integers?
(b) n_1, n_2, n_3 and n_4 are nonegative integers and n_3 is larger than 7?