

FINAL EXAM
Math 2513
12-13-04

1. (15 points) Use induction to prove that $\sum_{k=0}^n 2k + 1 = (n + 1)^2$ for every $n \in \mathbb{N}$.
2. (15 points) (a) Let A and B be sets. Give the definition of the set difference $A - B$.
(b) Use basic definitions to show that if A and B are disjoint sets then $A - (A - B) = \emptyset$.
3. (5 points) Draw a schematic diagram of a directed graph which has (directed) paths of length 2 and 3 but no (directed) path of length 5.
4. (20 points) Let A be the set of all bit strings of length 10.
(a) How many elements does A have?
(b) Does A have more than 10^{10} subsets with exactly four elements?
(c) Let C be the subset of A consisting of those 10-strings which contain a "100100" substring. How many elements does C have?
(d) Let D be the subset of A consisting of those 10-strings which contain a "100100" substring or start with two successive 1's. Determine $|D|$.
(e) How many 10-strings contain a "100100" substring or a "00000" substring?
5. (20 points) (a) Let X and Y be sets and $f : X \rightarrow Y$ be a function. Define what it means for f to be one-to-one.
(b) Give an example of a function which is not one-to-one.
(c) Give an example of a function which is one-to-one.
(d) If X has n elements and Y has k elements, then how many one-to-one functions from X to Y are there?
(e) Let $X = \{x_1, x_2, x_3, x_4\}$ and $Y = \{y_1, y_2, y_3, y_4, y_5\}$. How many one-to-one functions $f : X \rightarrow Y$ are there that satisfy $f(\{x_1, x_2\}) \subseteq \{y_1, y_2\}$?
6. (15 points) Let r_1 and r_2 be rational numbers where $r_2 \neq 0$. Show that $2r_1 + \frac{r_1}{r_2}$ is rational.
7. (10 points) Let $A = \{1, 2, 3\}$. (a) How many different relations are there on A ? (b) Give an example of a relation on A which contains $(1, 3)$ but is neither symmetric nor anti-symmetric. (c) Give an example of a relation on A that is both symmetric and anti-symmetric.