Math 2513
Example 10, page 89

In this course students are strongly encouraged to learn to write mathematical proofs using everyday, common-sense language, and not relying on the use of arcane logical symbols. Here is a proof of the second of DeMorgan's laws which avoids using logical symbols. Compare this with the proof of Example 10 on page 89 of Rosen's book.

EXAMPLE 10. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

proof: We will prove that these two sets are equal by showing that each is a subset of the other.

Suppose that x is an element of $\overline{A \cap B}$. By the definition of complementation this means that $x \notin A \cap B$. For x to be an element of $A \cap B$ we must have $x \in A$ and $x \in B$. So, since x is not an element of $A \cap B$ then either $x \notin A$ or $x \notin B$. Using the definition of complementation, this means that $x \in \overline{A}$ or $x \in \overline{B}$. Therefore $x \in \overline{A} \cup \overline{B}$ by the definition of union. This shows that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$.

Now suppose that $x \in \overline{A} \cup \overline{B}$. From the definition of union it follows that $x \in \overline{A}$ or $x \in \overline{B}$. Thus $x \notin A$ or $x \notin B$ by the definition of complementation. If x were an element of $A \cap B$ then x would be an element of both A and B which is impossible. So we conclude that $x \notin A \cap B$. By the definition of complementation this means that $x \in \overline{A \cap B}$. This shows that $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$. Since we have shown that each set is a subset of the other, the two sets are equal, and DeMorgan's second identity is proved. □