Problem. Let $a$, $b$ and $c$ be positive integers. Prove that if $c$ divides $a$ then $c$ divides $ab$.

Solution: The statement to be proved is a simple implication whose hypothesis is "$c$ divides $a$" and whose conclusion is "$c$ divides $ab$". We will give a direct proof. This means that we will assume that $c$ divides $a$ and then use logical inference to show that $c$ divides $ab$. The key definition is: If $m$ and $n$ are integers and $m \neq 0$ then $m$ divides $n$ if there is an integer $k$ such that $n = mk$.

Proof: Let $a$, $b$ and $c$ be integers. Assume that $c$ divides $a$. By definition (of divides) this means that $c \neq 0$ and that there is an integer $n$ such that $a = cn$. Then $ab = (cn)b = c(nb)$, and since $nb$ is an integer (the product of integers is always an integer) it follows (by the definition of divides) that $c$ divides $ab$. \qed