Problem. Let $A$ and $B$ be sets. In each problem a proposition $P$ is given. In each case write as directly as possible a statement for the negation $\neg P$.

(1) $x$ is an element of $A$, and $A$ and $B$ are disjoint.
(2) There exists an element of $A$ which is not an element of $B$.

SOLUTION:

(1) By common sense reasoning the negation of the proposition ”$x$ is an element of $A$, and, $A$ and $B$ are disjoint” is the proposition ”either $x$ is not an element of $A$, or, $A$ and $B$ are not disjoint”. Using symbols we could write this negation as ”$x \notin A$ or $A \cap B \neq \emptyset$”. (Remember that two sets are ”disjoint” if their intersection equals the emptyset.)

(2) The negation of the proposition ”there exists an element of $A$ which is not an element of $B$” is the proposition ”every element of $A$ is an element of $B$”. Note that this negation can be written more succinctly as ”$A \subseteq B$” (using the definition of subset).