Class Problem
Math 2513
February 24, 2005

PROBLEM. Let A and B be sets. In each problem a proposition $\mathcal{P}$ is given. In each case write as directly as possible a statement for the negation $\neg\mathcal{P}$.
(1) x is an element of A, and A and B are disjoint.
(2) There exists an element of A which is not an element of B.

SOLUTION:
(1) By common sense reasoning the negation of the proposition "x is an element of A, and, A and B are disjoint" is the proposition "either x is not an element of A, or, A and B are not disjoint". Using symbols we could write this negation as "$x \notin A$ or $A \cap B \neq \emptyset$". (Remember that two sets are "disjoint" if their intersection equals the emptyset.)
(2) The negation of the proposition "there exists an element of A which is not an element of B" is the proposition "every element of A is an element of B". Note that this negation can be written more succinctly as "$A \subseteq B$" (using the definition of subset).