1. (15 points) Consider the set identity $A - B \subseteq A$.
   (a) Write down complete statements of the two definitions which play central roles in this identity.
   (b) Prove the identity.

2. (10 points) In one or two sentences describe the standard general strategy which you would use to prove that two given sets are equal.

3. (15 points) Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$ be the function described by the rule $f(n) = n^2 + 1$. (a) What are the domain, codomain and range of this function $f$? (b) Show (by example) that $f$ is not injective. (c) Determine $f(S)$ if $S = \{-1, 0, 1, 2, 3\}$.

4. (15 points) Let $A$ and $B$ be sets such that $A \subseteq B$. Show that $A \cup B \subseteq B$.

5. (10 points) Let $A$ and $B$ be two non-empty sets. (a) Describe the Cartesian product $A \times B$. (b) Let $P : A \times B \rightarrow B$ be the function which maps $(a, b)$ to $b$. Show that $P$ is onto.

6. (10 points) Let $f$ and $g$ be functions from $\mathbb{R}$ to $\mathbb{R}$ given by $f(x) = 3x - 2$ and $g(x) = ax + b$ where $a$ and $b$ are constants. (a) Determine an equation for $g \circ f(x)$ (b) Find values for $a$ and $b$ such that $g \circ f(x) = x$ for every $x \in \mathbb{R}$. Then determine $f \circ g(x)$ when these values of $a$ and $b$ are used.

7. (10 points) Explain why the function $h : \{a, b, c, d\} \rightarrow \{1, 2, 3, 4\}$ defined by $h(a) = 2$, $h(b) = 4$, $h(c) = 3$, and $h(d) = 1$ is a bijection.

8. (15 points) Let $A$, $B$ and $C$ be sets. Prove that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$. 