1. (15 points) Use a proof by contradiction to show that the sum of a rational number and an irrational number is irrational.

**ANSWER:** This problem was discussed in class.

2. (10 points) Use the binomial theorem to determine the $x^6y^3$ term in the expansion of $(2x - 3y)^9$.

**ANSWER:** $C(9, 3)(2x)^6(-3y)^3 = -145152x^6y^3$

3. (20 points) Determine the number of bit strings which satisfy each of the following and clearly name any counting principles that are used:
   (a) The strings of length 8 which begin with 4 zeros and end with 101?
   (b) The strings of length 8 which begin with 4 zeros or end with 101?
   (c) The strings of length 8 which have exactly 4 zeros.

**ANSWER:** (a) 2, (b) $2^4 + 2^5 - 2 = 46$ using inclusion/exclusion, (c) $C(8, 4) = 70$

4. (15 points) (a) How many different strings of length 5 can be made from the letters in "BOBBY"
   (b) How many different strings of length 4 can be made from the letters in "BOBBY"

**ANSWER:** (a) 20, (b) 20

5. (20 points) Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4\}$.
   (a) How many functions from $A$ to $B$ are there?
   (b) How many 1-1 functions from $A$ to $B$ are there?
   (c) How many onto functions from $A$ to $B$ are there?
   (d) How many functions $f$ from $A$ to $B$ are there which satisfy the condition that there are exactly two elements $x$ in $A$ with $f(x) = b_3$?

**ANSWER:** (a) $4^3 = 64$, (b) $4 \cdot 3 \cdot 2 = 24$, (c) 0, (d) $C(3, 2) \cdot 3 = 9$

6. (20 points) Let $R$ be the relation on $\mathbb{N}$ given by $R = \{(m, n) \mid m, n \in \mathbb{N} \text{ and } (m - n)(m - 1) = 0\}$. Explain what each of the following means, and then determine whether this relation $R$ satisfies it:
   (a) $R$ is reflexive. (b) $R$ is symmetric. (c) $R$ is anti-symmetric. (d) $R$ is transitive.

**ANSWER:** (a) YES, (b) NO, (c) YES, (d) YES