1. (15 points) Prove that there are infinitely many prime integers.
2. (20 points) Use the Principle of Mathematical Induction to prove that: \(2(1 + 3 + 3^2 + \cdots + 3^n) = 3^{n+1} - 1\) for every positive integer \(n\).

3. (10 points) State the result that you are asked to prove in problem 2 using summation notation.
4. (20 points) Determine whether each of the following is true or false and then prove your assertion.
(a) If $a$, $b$ and $c$ are positive integers such that $c$ divides $a - b$ then $c$ divides $a$.
(b) If $a$, $b$ and $c$ are positive integers such that $c$ divides $a$ and $c$ divides $b$ then $c$ divides $a - b$.
(c) There are no integers $a$ and $b$ such that $a^2 + 4b^2 = 26$. 
5. (10 points) Let $A$ and $B$ be sets such that $A \subseteq B$. Show that $B - A = \emptyset$. 
6. (15 points) Illustrate the Euclidean algorithm by using it to find the greatest common divisor of 42 and 140.

7. (10 points) (a) Find at least two positive integers $n$ which satisfy $n \equiv 7 \pmod{11}$.
   (b) Find at least two negative integers $n$ which satisfy $n \equiv 4 \pmod{23}$.
   (c) Find at least two integers $n$ which satisfy both $n \equiv 7 \pmod{11}$ and $n \equiv 4 \pmod{23}$. 