Comments on some problems from HomeWork #7
Math 2513
10/14/04

Section 3.2, Problem 6: The answers are:

a) \{10, 7, 4, 1, -2, -5, \ldots\}

b) \{1, 3, 6, 10, 15, 21, \ldots\}

c) \{1, 5, 19, \ldots\}

d) \{1, 1, 1, 2, 2, 2, 3, \ldots\}

e) \{1, 2, 3, 5, 8, 13, \ldots\}

f) \{1, 3, 7, 15, 31, 63, \ldots\}

g) \{1, 2, 4, 8, 11, 33, \ldots\}

h) \{1, 2, 2, 2, 3, 3, \ldots\}

Section 3.2, Problem 14: The answers are:

a) 16

b) 84

c) 176/105

d) 4

Section 3.3, Problem 10: Prove that \(\sum_{i=1}^{n} i \cdot i! = (n+1)! - 1\).

We will use mathematical induction taking \(P(n)\) to be the statement that \(\sum_{i=1}^{n} i \cdot i!\) equals \((n+1)! - 1\).

**Basis Step.** When \(n = 1\) we have \(\sum_{i=1}^{n} i \cdot i! = 1 \cdot 1! = 1\) and \((n+1)! - 1 = 2! - 1 = 1\). This shows that \(P(1)\) is true.

**Inductive Step.** Let \(k\) be a positive integer and assume that \(P(k)\) is true. This means that \(\sum_{i=1}^{k} i \cdot i! = (k+1)! - 1\). We have

\[
\sum_{i=1}^{k+1} i \cdot i! = \left(\sum_{i=1}^{k} i \cdot i!\right) + (k+1) \cdot (k+1)! = ((k+1)! - 1) + (k+1) \cdot (k+1)! = (k+2) \cdot (k+1)! - 1 = (k+2)! - 1.
\]

This shows that \(P(k+1)\) is true. Therefore we have proven the inductive step, and the principle of mathematical induction allows us to conclude that \(\sum_{i=1}^{n} i \cdot i! = (n+1)! - 1\) for every positive integer \(n\).

\[\square\]

Section 3.3, Problem 52: The Basis Step is correct but there’s a mistake in the fourth sentence of the Inductive Step. It’s not necessarily true that \(\max(x-1, y-1) = k\), and the problem is that we don’t know that \(x-1\) and \(y-1\) are positive integers, so the inductive hypothesis cannot be applied.