

Math 2433
Sample Exam 1
Fall 2021

Name:

PROBLEM 1. (*20 points*) Let $\mathbf{a}$ be the vector represented by the arrow starting at $P = (2, 3, -5)$ and ending at $Q = (0, 4, -7)$. Let $\mathbf{b}$ be a vector with length 4 which forms an angle of $\pi/4$ with $\mathbf{a}$.

- (a) Express $\mathbf{a}$ in terms of the coordinate vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.
- (b) Find the two unit vectors which are parallel to $\mathbf{a}$.
- (c) Determine $\mathbf{a} \cdot \mathbf{b}$.

PROBLEM 2. (*20 points*) Let $\mathbf{a} = \langle 1, 0, 2 \rangle$ and $\mathbf{b} = \langle -2, 1, 3 \rangle$.

- (a) Determine the magnitudes of $\mathbf{a}$ and $\mathbf{b}$ and the cosine of the angle between the two vectors.
- (b) Find two unit vectors that are orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.
- (c) What is the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$?

PROBLEM 3. (10 points) Determine whether or not the four points $(1, 1, 1)$, $(3, -1, 0)$, $(-1, 0, 2)$, $(7, 5, -2)$ and are coplanar in $\mathbb{R}^3$.

PROBLEM 4. (20 points) Let $\mathbf{u} = \langle -6, 1, 3 \rangle$ and $\mathbf{v} = \langle 4, 0, -2 \rangle$.

- (a) If $\mathbf{u} = \vec{PQ}$ and $Q = (10, -2, 7)$ then what is P ?
- (b) Determine the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.
- (c) Find the two unit vectors that are parallel to $\mathbf{v}$.
- (d) Determine the vector projection $\text{proj}_{\mathbf{u}}(\mathbf{v})$ of $\mathbf{v}$ onto $\mathbf{u}$.
- (e) (bonus) If the vector projection $\text{proj}_{\mathbf{v}}(\mathbf{u})$ equals $\mathbf{v}$ what does that say about the relationship between $\mathbf{u}$ and $\mathbf{v}$.

PROBLEM 5. (15 points) Three curves are described by parametrizations

$$C_1 : x = t, y = t^2 - 1, \quad C_2 : x = t^2, y = t^4 - 1, \quad C_3 : x = \cos(t), y = \cos^2(t) - 1 .$$

Draw separate pictures of the three curves and describe how they are related yet different.

PROBLEM 6. (20 points) Consider the curve described by the parametric equations $x = t - t^2$, $y = t - t^3$.

- (a) Does the curve pass through the point $(-2, 3)$? Explain.
- (b) Find all points on the curve where the tangent line to the curve has slope 5. (Giving t -values is sufficient.)
- (c) Determine d^2y/dx^2 .

PROBLEM 7. (20 points) An object in motion in the plane is located at $(x, y) = (2t^3 + 3t^2 - 12t + 7, t^2 - 1)$ at time t (where $-\infty < t < \infty$). Let C be the curve that it traces out.

- (a) Determine any points where C crosses the x -axis.
- (b) Find an equation for the line which is tangent to C at the point where $t = 2$.
- (c) For which values of t is the object moving upward?
- (d) For which values of t is the object moving to the right?
- (e) Use your answers to (c) and (d) to draw a rough picture of C .
- (f) The curve C has one point where it crosses itself. Find the t -values for that point.