Lines and Planes - RECAP

Lines in 3-space

Each line l in xyz-space has a parametrization:

 $L: \begin{cases} x = at + x_0 \\ y = bt + y_0 \\ z = ct + z_0 \end{cases}$ 

for some constants a,b, c and xo, yo, 30. The vector To= (a,b,c) is called a direction vector for & (it is parallel to l). The point on I where t=0 is (xo, yo, Zo).

orwrite this as: Ax+By+Cz=-D

Planes in 3-space

Every planep in 3-space has an equation of the form:

 $p: A \times + B_{y} + C_{z} + 0 = 0$ 

(called a "linear equation with 3 variables".)

The vector  $\tilde{N} = \langle A, B, C \rangle$  is perpendicular to p, it is called a normal vector for p.

Example : Are these two lines parallel? Ji=direction vector for l, = <-2, 1, -37 e Trese are The z = Direction vector for lz = <6, -3, 9) For allel vectors conclude li and lz are parallel. 3di

note well: If I is a direction vector for I then any vector parallel to d is also a direction vector for l. In this example (-2, 1, -3) is a direction vector for both l, and lz.

Follow-Up Question Does la equal li?

(0,0,0) is a point on  $l_1$ . Is it also on  $l_2$ ? YES: The equations  $\begin{cases} Cf+4 = 0\\ -3t-2 = 0\\ qf+6 = 0 \end{cases}$  have a solution  $t = \frac{-2}{3}$ 

conclude lz=l [Parallel lines with a point in common must be equal.] Planes in 3-space

Every planep in 3-space has an equation of the form:

 $P: A_{X+}B_{Y+}C_{z}+V=0$ 

(called a "linear equation with 3 variables".)

The vector  $N = \langle A, B, C \rangle$  is perpendicular to p, it is called a normal vector for p.

notewell: If N is a normal vector for p then any vector parallel to N is also a normal vector for p. Example Consider the plane p: 3x-2y-2+6=0. () Is the point (-1,-1,-1) on p? No () Is the point (-1,-1,-1) on p? No ()  $N = \langle 3, -2, -1 \rangle$ (2) Find some points that are on p. or use a hit or miss approach: E.S. - Try  $x=2, y=-1 \implies 3(2)-2(-1)-2+6=0 \implies 2= 14$ 

=) (2,-1, 14) is on the plane p.

(3) Equations for some planes parallel to pere: 3x-2y-2-6=0 these all have normal 3x-2y-2-6=0 rector (3,-2,-1). 3x-2y-2+1=0 (Parallel planes most have porallel normal 3x-2y-2=0 this are goes thru origin!

(4) How does the plane p: -12x+8y+4z-24=0 compare with p? answer: p=p, (just multiply equation for p by-4)

Strategy for finding the equation of a plane p () Find a point P=(x0, 40, 20) on p. (ii) Find a vector N = < A, B, c> perpendicular to p ("normal vector"). Than an equation for the plane is  $p: A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$ 

explain If a point Q = (x, y, z) is in the plane p then PQ = <x-x0, 4-40, 2-20> is I to N. So  $O = \tilde{N} \cdot \tilde{PQ} = A(x-x_0) + B(y-y_0) + C(z-z_0).$ 

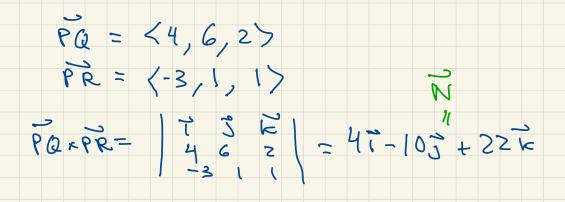
example : A plane p is parallel to the plane Pi: 3x+7y-5z=0 and contains (-1,-2,3). Find an equation for p. () P = (-1, -2, 3) is on p

D NI=<3,7,-5> is a normal vector for both p1 and p.

So p: 3(x+1)+7(y+2) - 5(z-3) = 0p: 3x+7y-5z+3z=0 Example :

Find an equation for the plane p which contains the three points P=(1,-1,3), Q=(5,5,5) and R=(-2,0,4).

(i) R = (-2,0,4) is a point on p (i) PaxPR is a normal vector for p



p: 4(x+2) - 10(y-0) + 22(2-4) = 0

4x-10y+222-80=0

2x-5y+112-40=0

Two lines in 3-space are either: 0 equal @ parallel but not equal B intersect in one point (4) skew note: In cases @ and @ there is a unique plane containing the two lines. Two planes in 3-space are either: D equal @ parallel but not equal 3 intersect in a line note: In 3, the cross product of normal vectors for the planes is a direction vector for the line of intersection. A line I and a plane p satisfy one of: O lis contained in p @ l is parallel to p but not contained in p (3) & intersects p in one point note: In Oor @, a direction vector for & will be perpendicular to a normal vector for p.