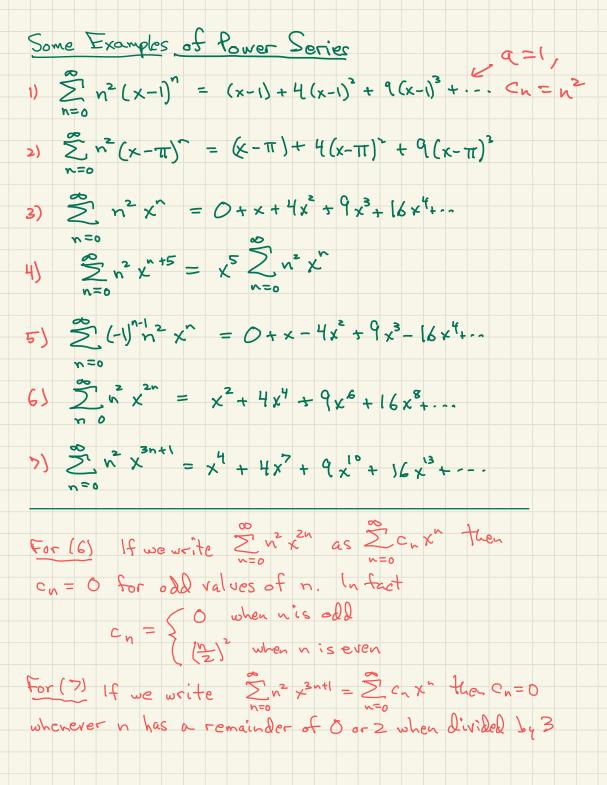
A series of the form $\sum_{n=0}^{3} C_{n} (x-a)^{n} = C_{0} + C_{1} (x-a) + C_{2} (x-a)^{2} + C_{3} (x-a)^{3} + \cdots$ is a "power scries centered at x=a". (Here a is a constant, x is a variable, and each cn is a number that does not depend on x.) When q=0 we have $\widetilde{\sum}_{n=0}^{\infty} C_n \chi^{\hat{n}} = C_0 + C_1 \chi + C_2 \chi^2 + C_3 \chi^3 + \cdots$ Each power series determines a Function $f(x) = \sum_{n=1}^{\infty} c_n (x-a)^n$ The domain of this function consists of all numbers x for which the series converges. This domain is always an interval centered at x=a, and t is called the interval of convergence of the power series. Example Geometric Series!! $f(x) = \sum_{n=0}^{1} x^n$ (power series where $c_n = 1$ for all n) This function has domain (f) = {x where |x(<1} = (-1,1) and we have a formula $f(x) = \frac{1}{1-x} = \sum_{n=0}^{1} x^n - 1 < x < 1$



Interval of Convergence I Every power series $\sum_{i=1}^{n} C_{n} (x-a)^{n} = C_{0} + C_{1} (x-a) + C_{2} (x-a)^{2} + \cdots$ has an associated radios of convergence R with the property that the series · converges when |x-a| < R, and · diverges when 1x-al>R. This means that the interval of convergence I of the power series is one of : (a-R,a+R), [a-R,a+R], (a-R,a+R], [a-R,a+R] open interval half-open interval closed interval Notice that each of these possible intervals has center (that is, milpoint) at x=a. Strategy for determining interval of convergence: 1) Use Ratio test tofind R. Use other tosts to Determine convergence at the two endpoints: x=a-R, x=a+R in the cases where R=0 or R=00, step 2 will be unnecessary.

Examples

1) $\sum_{n=0}^{\infty} x^n$, R=1, I=(-1,1) (geometric) (2) $\sum_{n=0}^{\infty} \frac{3^n}{n} \times n$. (1) Use Ratio test: $a_n = \frac{3^n}{n} \times n$ $\frac{|a_{n+1}|}{|a_{n}|} = \frac{3^{n+1}|x|^{n+1}}{n+1} \cdot \frac{n}{3^{n}|x|^{n}} = 3|x| \cdot \frac{1}{1+1/n} = 3|x| = 1$ So, series converges when 31x1<1 (means 1x1</3) and diverges when 31x1>1 (means 1x1>1/3) (ii) The endpoints are x = -1/3 and x = 1/3. check $x = \frac{1}{3}$: $\sum_{n=0}^{\infty} \frac{3^{n}}{n} (\frac{1}{3})^{n} = \sum_{n=0}^{\infty} \frac{1}{n}$ diverges. PS check $x = -\frac{1}{3}$ $\sum_{n=0}^{\infty} \frac{3^n}{n} \left(-\frac{1}{3}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{n}$ converges. AST $I = [-\frac{1}{3}, \frac{1}{3}] / R = (1_3)$ conclude : $\frac{(n+1)!}{n!} = \frac{(n+1)n!}{n!} = n+1$ $\exists \sum_{n=0}^{\infty} \frac{n!}{3^n} x^n .$ $\frac{|a_{n+1}|}{|a_{n}|} = \frac{(n+1)!}{3^{n+1}} \frac{3^{n}(x)^{n+1}}{n!} = \frac{n+1}{3}|x| \xrightarrow{n \to \infty} \infty$ conclude The series always diverges when x ≠ 0. So R=0, I= 203 case (iii) applies here . So power series converges only when X=O.

