Quiz 5-5 Answers

#1 The integral from 1 to 2 of 1/(x^2-1) is an improper integral. **True.**

The integrand $\frac{1}{x^2-1}$ is undefined when $x=1$ or $x=-1$.
So the integral is improper and

$$\int_1^2 \frac{1}{x^2-1}\,dx = \lim_{a\to 1+} \int_1^2 \frac{1}{x^2-1}\,dx$$

#2 The integral from 1 to 2 of the function f(x)=1/(x^2+1) is an improper integral. **False.**

$f(x) = \frac{1}{x^2+1}$ is continuous on the interval $[1,2]$ and
we can calculate $\int_1^2 \frac{1}{x^2+1}\,dx = \arctan(2) - \arctan(1)$

#3 The region in the first quadrant which is below y=1/x^3 and to the right of x=1 has finite area. **True.**

This region has area

$$\int_1^\infty \frac{1}{x^3}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x^3}\,dx = \lim_{b\to\infty} -\frac{1}{2}\frac{1}{x^2}\Big|_1^b$$

$$= \lim_{b\to\infty} -\frac{1}{2b^2} + \frac{1}{2} = \frac{1}{2}$$

#4 The region in the first quadrant which is below the graph of y=1/x^3 and between the vertical lines x=0 and x=1 has finite area. **False**

$$\int_0^1 \frac{1}{x^3}\,dx = \lim_{a\to 0+} \int_a^1 \frac{1}{x^3}\,dx = \lim_{a\to 0+} -\frac{1}{2x^2}\Big|_a^1$$

$$= \lim_{a\to 0+}\left(-\frac{1}{2} + \frac{1}{2a^2}\right) = \infty$$