Math 2423
Quiz 4-26 (answers)

Problem 1-1:
One of the following quadratic polynomials is irreducible. Which one is it?
(a) $4x^2 + 4x + 1$
(b) $2x^2 - x - 6$
(c) $3x^2 + 6x + 4$

ANSWER: (c)
The discriminants for these quadratics are: (a) $4^2 - 4(1)(4) = 0$; (b) $(-1)^2 - 4(2)(-6) = 49$; (c) $6^2 - 4(3)(4) = -12$.
Observe that the quadratic in (a) is a perfect square $4x^2 + 4x + 1 = (2x + 1)^2$, the quadratic in (b) factors as $2x^2 - x - 6 = (2x - 3)(x + 2)$, whereas the quadratic in (c) cannot be factored.

Problem 1-2:
One of the following quadratic polynomials is irreducible. Which one is it?
(a) $9x^2 + 6x + 1$
(b) $2x^2 - x + 6$
(c) $x^2 + 6x - 4$

ANSWER: (b)
The discriminants for these quadratics are: (a) $6^2 - 4(1)(9) = 0$; (b) $(-1)^2 - 4(2)(6) = -47$; (c) $6^2 - 4(1)(-4) = 52$.
Observe that the quadratic in (a) is a perfect square $9x^2 + 6x + 1 = (3x + 1)^2$, the quadratic in (b) cannot be factored, whereas the quadratic in (c) factors as $x^2 + 6x - 4 = (x + 3 - \sqrt{13})(x + 3 + \sqrt{13})$.

Problem 2-1:
The rational function $\dfrac{3x-1}{(x+1)(x-3)}$ can be expressed with the form $\dfrac{A}{x-3} + \dfrac{B}{x+1}$ where A and B are constants. What is the value of A?

ANSWER: $A = 2$ and $\dfrac{3x-1}{(x+1)(x-3)} = \dfrac{2}{x-3} + \dfrac{1}{x+1}$.

Problem 2-2:
The rational function $\dfrac{x-7}{(x+1)(x-3)}$ can be expressed with the form $\dfrac{A}{x-3} + \dfrac{B}{x+1}$ where A and B are constants. What is the value of A?

ANSWER: $A = -1$ and $\dfrac{x-7}{(x+1)(x-3)} = \dfrac{-1}{x-3} + \dfrac{2}{x+1}$.

Problem 2-3:
The rational function $\dfrac{5x-3}{(x+1)(x-3)}$ can be expressed with the form $\dfrac{A}{x-3} + \dfrac{B}{x+1}$ where A and B are constants. What is the value of A?

ANSWER: $A = 3$ and $\dfrac{5x-3}{(x+1)(x-3)} = \dfrac{3}{x-3} + \dfrac{2}{x+1}$.
