Math 2423, Spring 2021
Quiz 4-12 (answers)
True/False

Problem 1:

- $0 \cdot \infty$ *is a determinate form*: FALSE
- $0 \cdot \infty$ *is an indeterminate form*: TRUE

To say that $0 \cdot \infty$ is an indeterminate form means that: *Just knowing that* $\lim_{x\to a} f(x) = 0$ *and that* $\lim_{x\to a} g(x) = \infty$ *is not enough information to tell what the limit of the product* $\lim_{x\to a} f(x)g(x)$ *might equal.* (NEITT)

Problem 2:

- $\displaystyle\lim_{x\to\infty} \frac{5x^3+1}{7x^2+3x+2} = 5/7$: FALSE
- $\displaystyle\lim_{x\to\infty} \frac{5x^3+1}{7x^3+3x+2} = 5/7$: TRUE

For the first statement:

$$\lim_{x\to\infty} \frac{5x^3+1}{7x^2+3x+2} = \lim_{x\to\infty} \frac{5x^3+1}{7x^2+3x+2} \cdot \frac{1/x^2}{1/x^2} == \lim_{x\to\infty} \frac{5x+1/x^3}{7+3/x+2/x^2} = \frac{\infty+0}{7+0+0} = \infty.$$

For the second statement:

$$\lim_{x\to\infty} \frac{5x^3+1}{7x^3+3x+2} = \lim_{x\to\infty} \frac{5x^3+1}{7x^3+3x+2} \cdot \frac{1/x^3}{1/x^3} = \lim_{x\to\infty} \frac{5+1/x^3}{7+3/x^2+2/x^3} = \frac{5+0}{7+0+0}.$$

(You could also use L'Hospital's Rule.)

Problem 3:

- *The graph of the function* $f(x) = \dfrac{xe^x+1}{e^x}$ *has a horizontal asymptote on the right*: FALSE
- *The graph of the function* $g(x) = \dfrac{e^x}{xe^x+1}$ *has a horizontal asymptote on the right*: TRUE

For the first statement

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{xe^x+1}{e^x} = \lim_{x\to\infty} x + \frac{1}{e^x} = \infty + 0 = \infty.$$

For the second

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{e^x}{xe^x+1} = \lim_{x\to\infty} \frac{1}{x+1/e^x} = 1/(\infty+0) = 0+\,.$$

This shows that the x-axis $y = 0$ is a horizontal asymptote for $g(x)$ on the right. (You could also use L'Hospital's Rule to calculate either of these limits).

Problem 4:

- $\displaystyle\lim_{t\to 0+} \frac{t^2+1}{\ln(t)} = 0$: TRUE
- $\displaystyle\lim_{t\to 0+} \frac{\ln(t)}{t^2+1} = 0$: FALSE

Use L'Hospital's Rule. For example

$$\lim_{t\to 0+} \frac{\ln(t)}{t^2+1} = \lim_{t\to 0+} \frac{1/t}{2t} = \lim_{t\to 0+} \frac{1}{2t^2} = \infty$$
