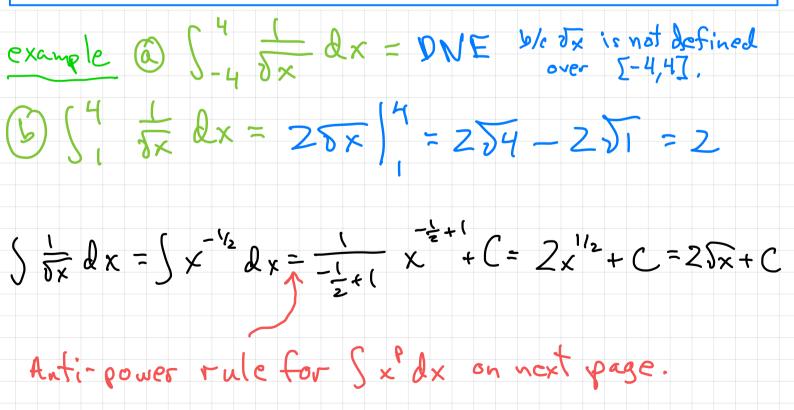
no limits of 1 integration. Notation for antiderivatives Write Jf(x) dx to denote the most general autiderivative of f(x), $e^{g} \int x dx = \frac{1}{2}x^2 + C$ we call Sf(x) dx the indefinite integral of f(x) with respect to x Stewart, page 331: Important!!!

You should distinguish carefully between definite and indefinite integrals. A definite integral $\int_a^b f(x) dx$ is a *number*, whereas an indefinite integral $\int f(x) dx$ is a *function* (or family of functions). The connection between them is given by Part 2 of the Fundamental Theorem: if *f* is continuous on [*a*, *b*], then

 \oslash

$$\int_{a}^{b} f(x) \, dx = \int f(x) \, dx \Big]_{a}^{b}$$



Some basic indefinite integrale

• $\int x^{p} dx = \frac{1}{p+1} x^{p+1} + C$ $if p \neq -1.$ T Why? b/c p+1 is undefined when p = -1Jsinxdx = -cosx + C •

 $\int \cos x \, dx = \sin x + C$ P

 $\int \sec^2(x) dx = \tan(x) + C$

Ssecxtarx &x = secx+C

General Rule of Thumb!

Differentiation is easy.

Integration is hard.

(But both have lots of important applications.)

Anti-power Rule is true because:

 $\frac{d}{dx} \left[x^{p} \right] = p x^{p-1} / p is a constant$ means $\int p x^{p-i} dx = x^{p} + C$ -> 11 PJxP-1dx $\left(\begin{array}{c} write \\ r=p-1 \end{array}\right)$ Divide by p to get $\int x^{e^{-t}} dx = \frac{1}{p} x^{p} + C$ e=L+1 $\int x^{r} dx = \frac{1}{r+1} x^{r+1} + C$ $\int x^{p} dx = \frac{1}{p+1} x^{p+1} + C$ Property for Indefinite Integrals $\int af(x) + bg(x) dx$ $= a \int f(x) dx + b \int g(x) dx$ (Linearity)

Every rule for differentiation gives a rule for anti-differentiation, but these are not always very useful. For example:

Qustiont Rule f'(x)g(x) - f(x)g'(x) $g(x)^{2}$ $\frac{Q}{Q \times \left[\frac{f(x)}{g(x)} \right]} =$ $\int \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} dx = \frac{f(x)}{g(x)} + C$ Complicated and not very useful. atall

Working with Mathematical Variables

A pair of variables - say u and v - can be cither "dependent" or "independent".

. The variables u and v are independent if changing one of the variables has no effect on the other variable. Inother words each variable is considered to be constant with respect to the other.

· The variables u and v are <u>dependent</u> if changing the value of one loss cause a change in the other variable. For example, if v = f(u) for some function f then u and v are dependent. Another example would be if a and b are variables satisfying the equation a² + b² = 1 then a and b are dependent variables.

Important:

Basic Standing Assumption If it is not explicitly stated that two variables are dependent then they are automatically considered to be independent.

Examples Jox2dx+Soldx $\int_{-3}^{0} x^{2} + \left| dx = \frac{1}{3}x^{3} + x \right|_{x=-3}^{0} = 0 - \left(\frac{1}{3}(-3)^{2} + (-3)^{2} + (-3)^{2}\right) = 12$ (2) $\int_{-3}^{0} x^{2} + 1 dt = (x^{2} + 1) + \int_{t=-3}^{0} = 3(x^{2} + 1)$ because x2+1 is constant with respect to t. (3) If X=Zt³ then dependent variables. $\int_{-3}^{0} x^{2} + 1 dt = \int_{-3}^{0} (2t^{3})^{2} + 1 dt = \int_{-3}^{0} 4t^{6} + 1 dt$ $= \frac{4}{7}t^{7} + t \Big|_{t=-3}^{0} = \frac{8769}{7}$ $(f) \int x^2 + (dt) = (x^2 + 1) t + C$ Conclusion: In either a definite integral Saffx) &x or an indefinite integral (f(x) dx, the "differential" dx is a required and important part of the notation.

Strive to be meticulous in your use of mathematical notation ...