.... continued example...

⑦ Find the range of the polynomial function

$$f(x) = x^4 - 132x^3 - 200x^2 + 0x^1 + 0x^0$$

degree 4 polynomial.

$\text{domain}(f) = \mathbb{R} = (-\infty, \infty) = (-\text{inf}, \text{inf})$

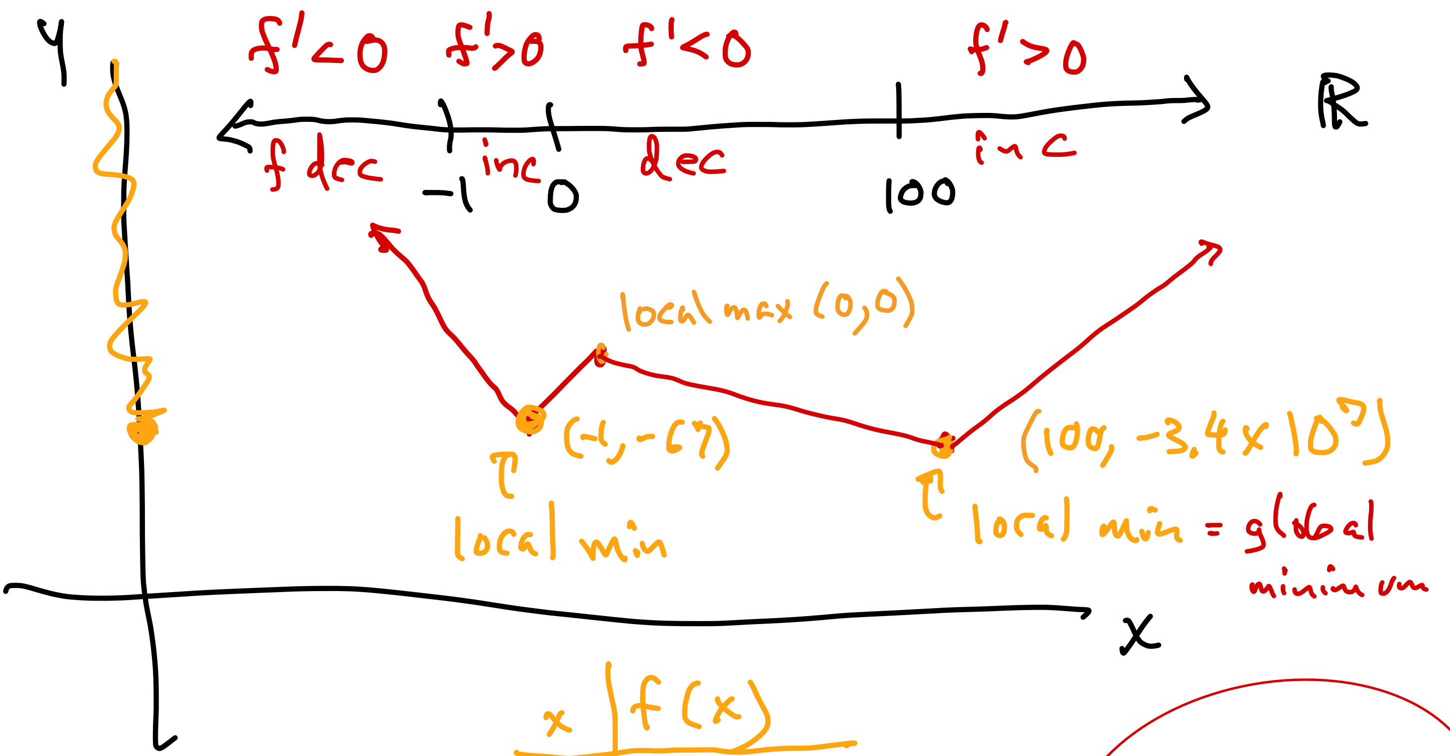
Let's sketch the graph.

$$f'(x) = 4x^3 - 396x^2 - 400x$$
$$= 4x(x^2 - 99x - 100)$$
$$= 4x(x+1)(x-100)$$

critical points

So $f'(x) = 0$ when $x = 0$, $x = -1$ or $x = 100$.

So $0, -1, 100$ are critical points for f



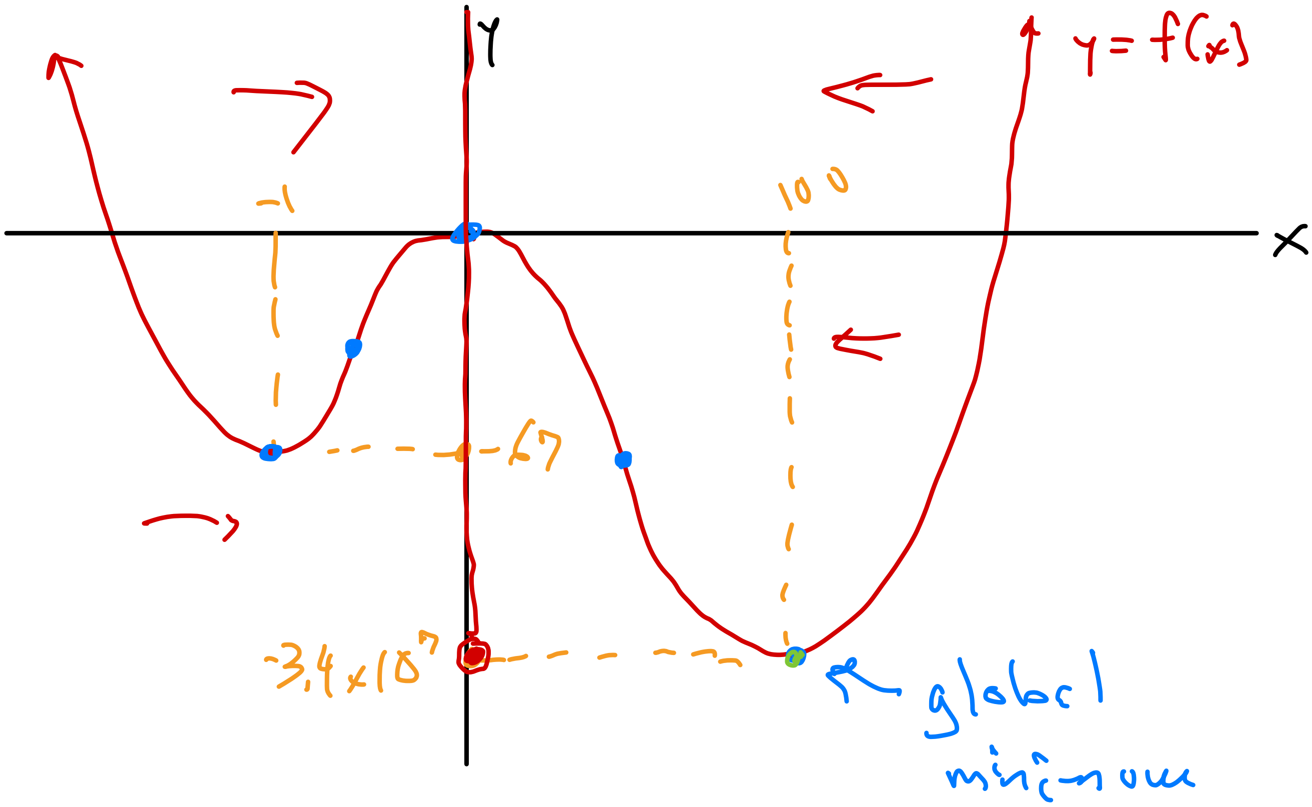
x	f(x)
-1	-67
100	-3.4×10^7
0	0

note: $f(100)$ actually equals -3.4×10^7

$\text{range}(f)$
$= (f(100), \infty)$
$= (-3.4 \times 10^7, \infty)$

With concavity information the graph of

$f(x) = x^4 - 132x^3 - 200x^2$ looks something like:



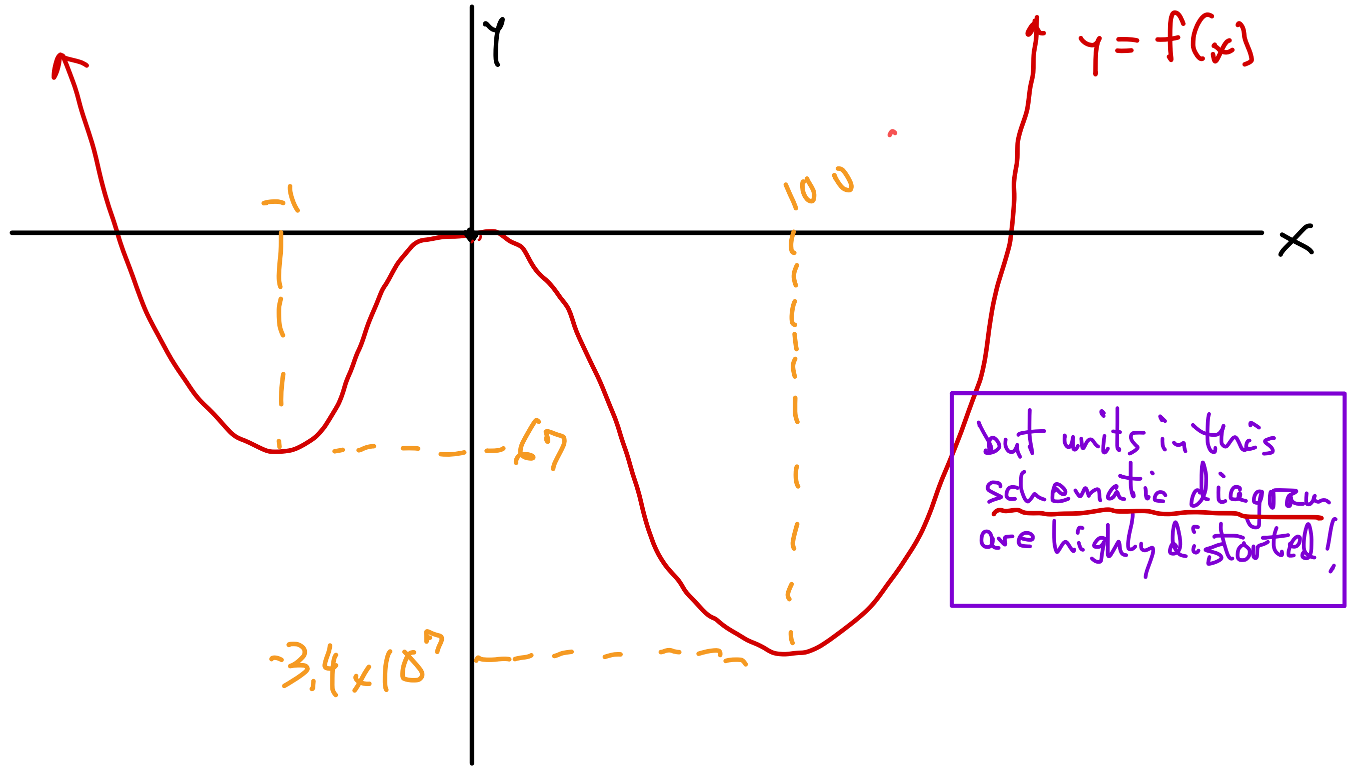
2 local mins
1 local max
2 points of inflection

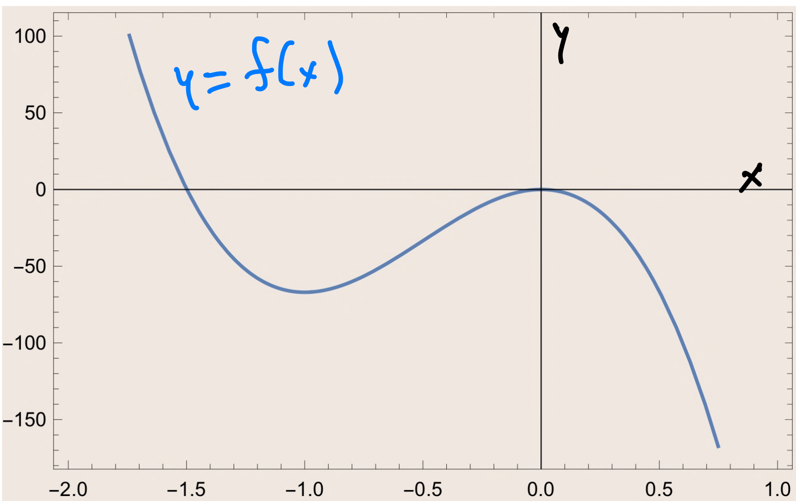
critical points

Range $= [-3.4 \times 10^7, \infty)$

With concavity information the graph of

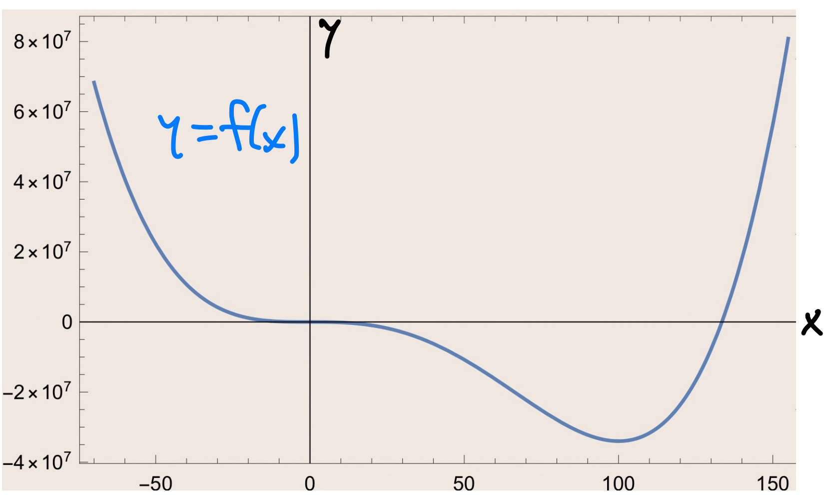
$f(x) = x^4 - 132x^3 - 200x^2$ looks something like:





Here's a graphing calculator picture with correct units and $-2 \le x \le 1$

Here's a graphing calculator picture with correct units and $-70 \le x \le 155$



In practice no single window can show all important features of the graph of $y = f(x)$.

from Stewart pages 224-225:

Definition A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.

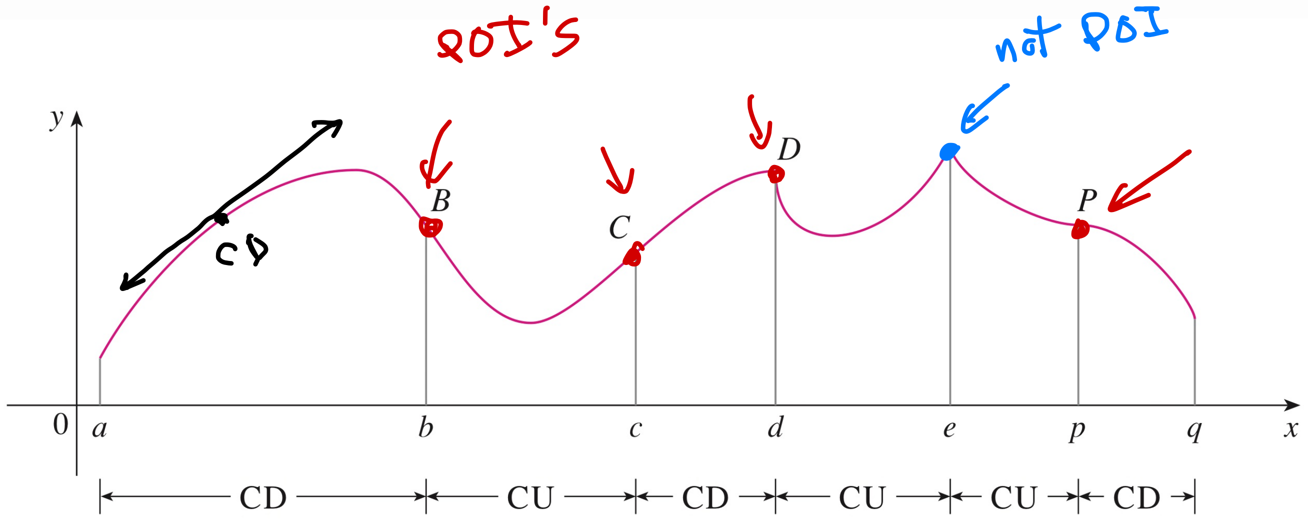


FIGURE 7

Definition If the graph of f lies above all of its tangents on an interval I, then it is called **concave upward** on I. If the graph of f lies below all of its tangents on I, it is called **concave downward** on I.

Theorem If $f(x)$ has a point of inflection at $(x_0, f(x_0))$ then either $f''(x_0) = 0$ or $f''(x_0) = \text{DNE}$.

Take Away:

Points where $f''(x_0) = 0$ or $f''(x_0) = \text{DNE}$ are the only candidates for points of inflection

Examples

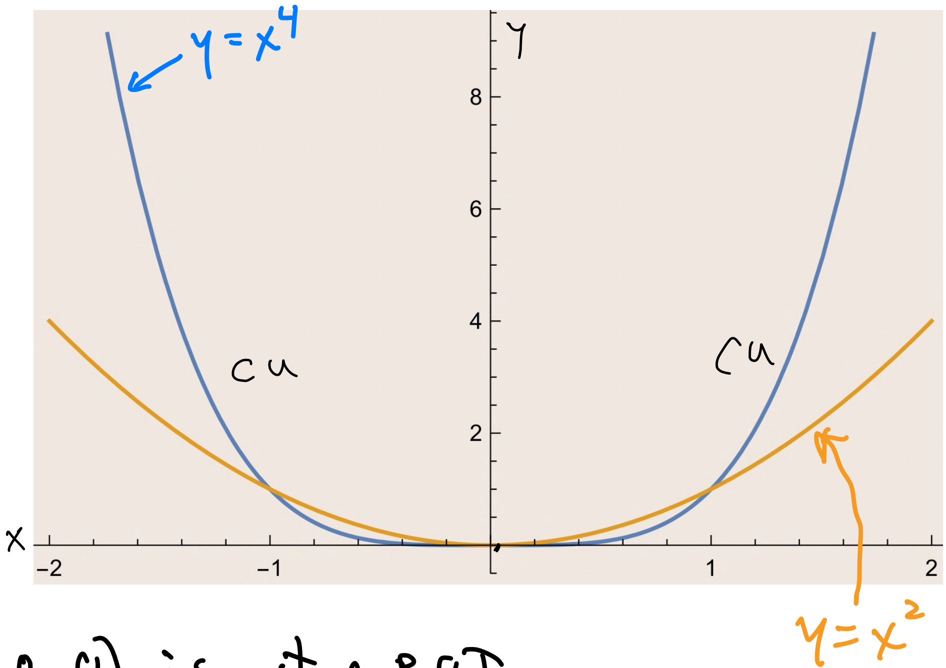
① $f(x) = x^4$ has no POI's but $f''(0) = 0$.

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$\underline{f''(0) = 0}$

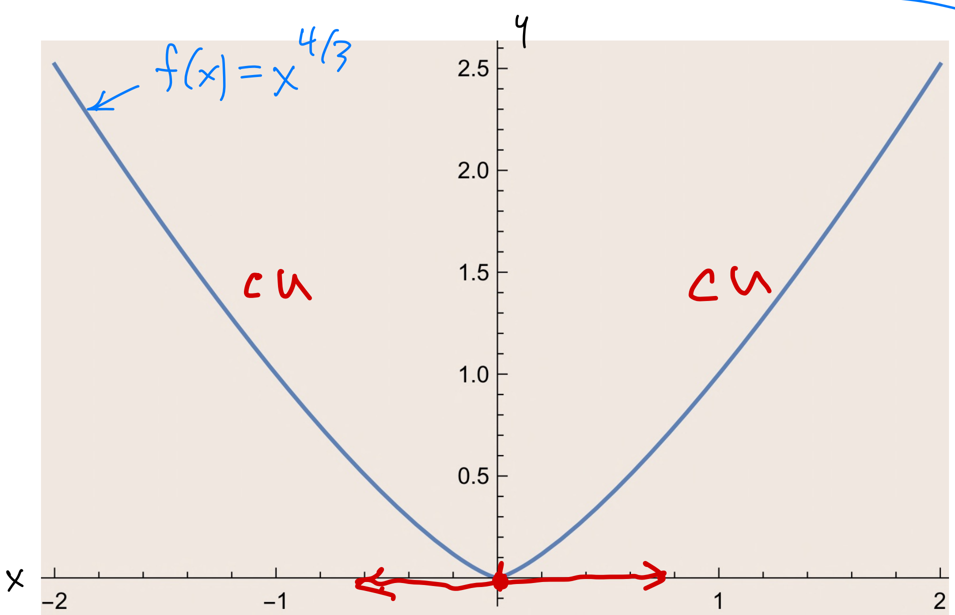
only POI candidate



but $(0,0)$ is not a POI.

② $f(x) = x^{4/3}$ has no POI's but $f''(0) = DNE$.

This is a power function not a polynomial



$f'(x) = \frac{4}{3} x^{1/3}$

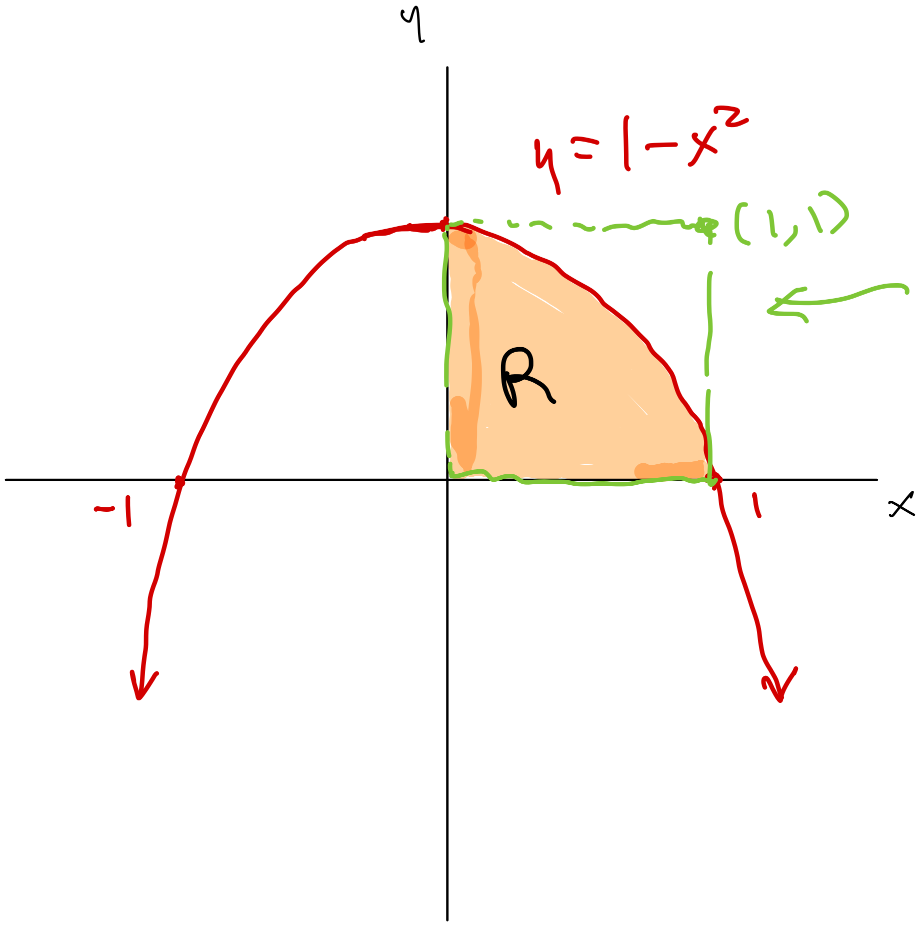
$f''(x) = \frac{4}{9} x^{-2/3}$

$= \frac{4}{9} \frac{1}{x^{2/3}}$

$f''(0) = DNE$ but $(0,0)$ is not a POI.

$f'(0) = 0$

Problem Find the area of the shaded region.



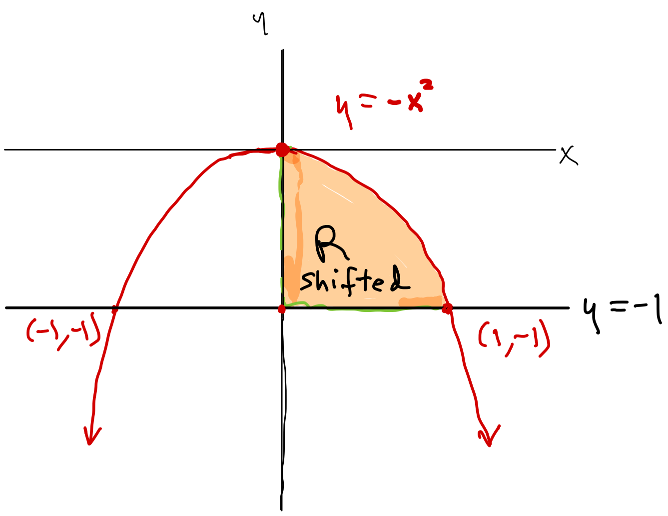
green square with side length 1

$\text{Area}(\text{green square}) = 1^2 = 1$

$$\text{area}(R) = ??$$

$$\begin{cases} \text{area}(R) \geq 0 \\ \text{area} \leq 1 \end{cases}$$

Important note: The area of a region in the plane is always positive!! For example if R is shifted down 1 unit, the area is unchanged!



Picture of R shifted down 1 unit.

General Problem (4.1)

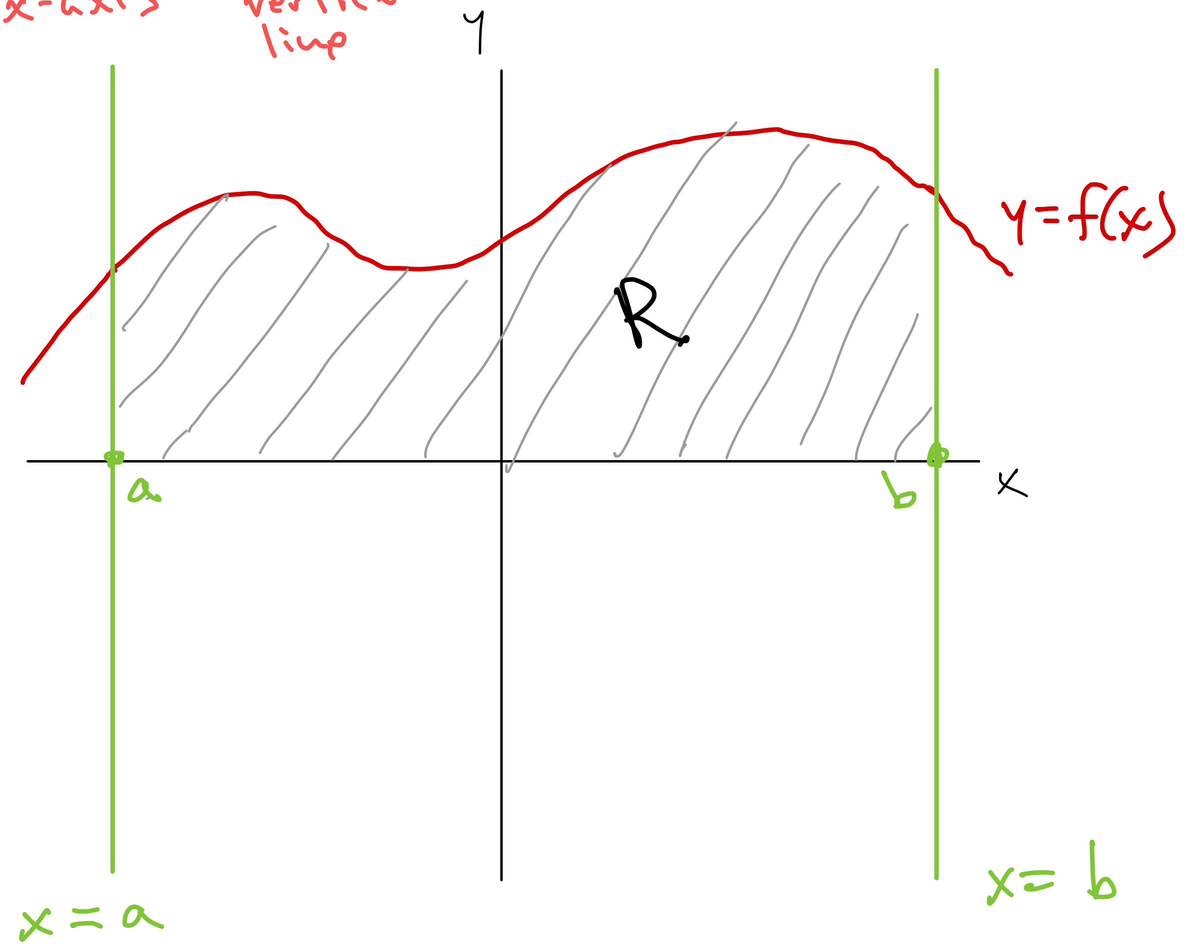
Suppose that $f(x) \geq 0$ for $a \leq x \leq b$. How can we approximate the area of the region R bounded by

describes the interval $[a, b]$

$y = f(x)$, $y = 0$, $x = a$, $x = b$?

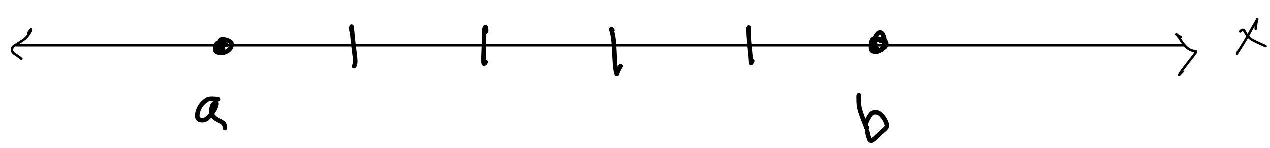
x-axis

vertical line



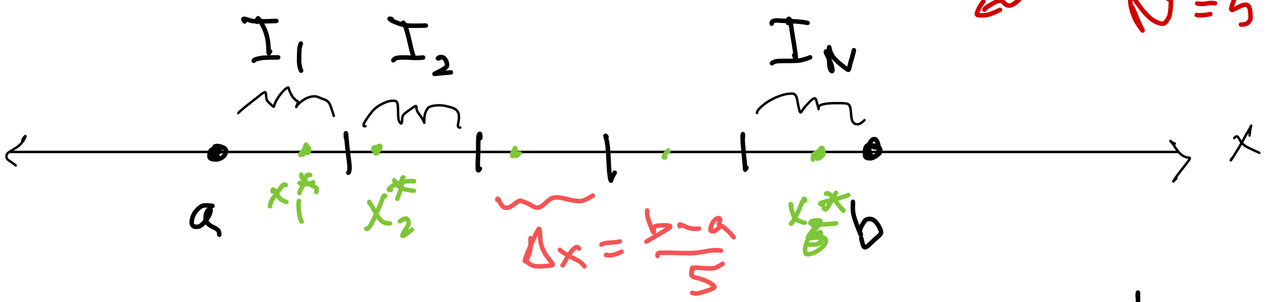
Start by subdividing the interval $[a,b]$ into a number N of subintervals all with the same length

picture for $N=5$



Call the subintervals $I_1, I_2, \ldots, I_N$

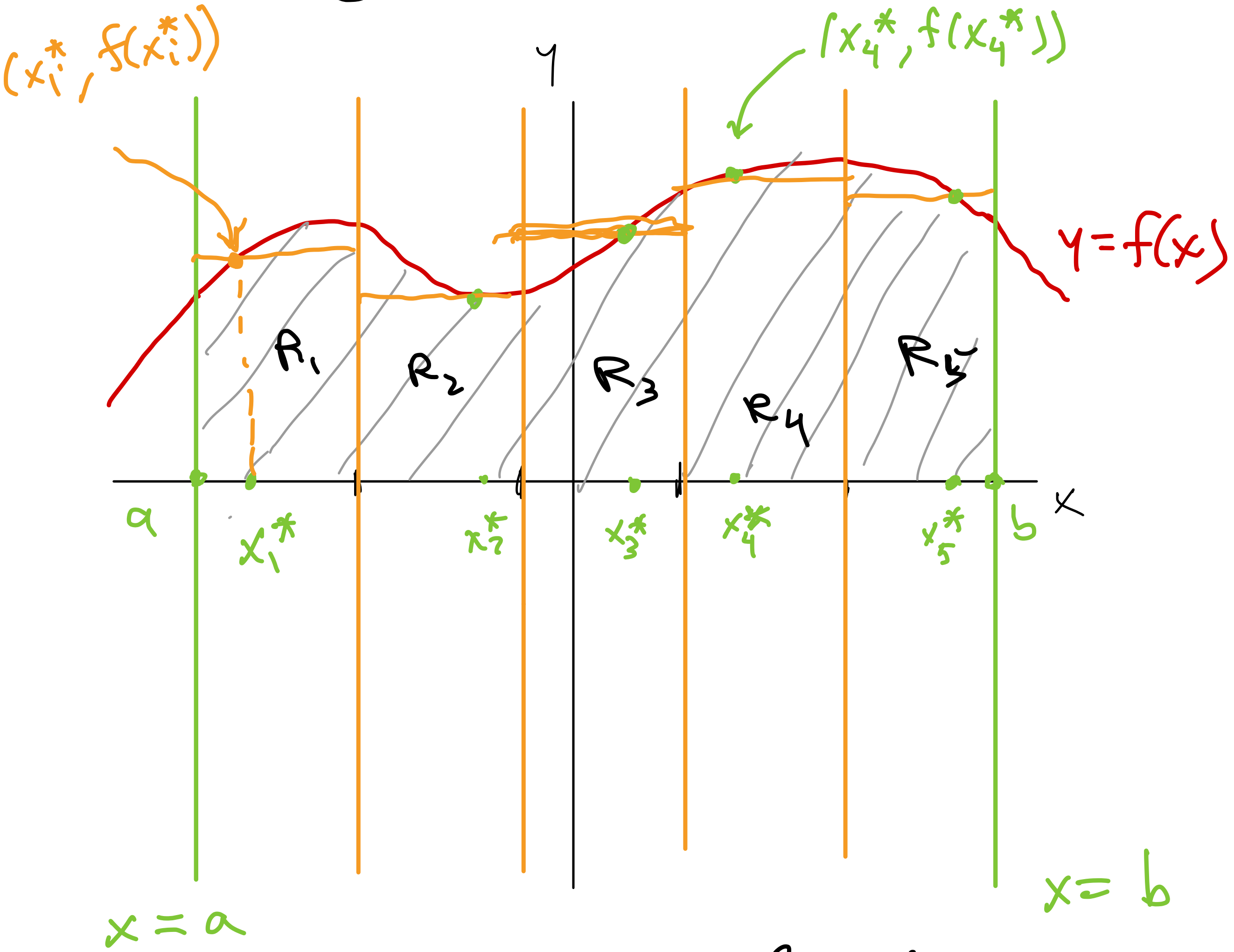
picture for $N=5$



The length of each subinterval is $\frac{b-a}{N} = \Delta x$.

Choose a point x_k^* in the k^{th} subinterval I_k.

Then construct a rectangle R_k above I_k with height $f(x_k^*)$ as shown below



$$\text{area}(R_k) = f(x_k^*)\,\Delta x$$

$\text{area}(R_1) + \text{area}(R_2) + \cdots + \text{area}(R_N)$

approximates the area R. Take $\lim_{N\to\infty}$ (sum of areas)