Math 2423
Team Assignment #2
due at class on Wednesday, September 13

Problem 1. Let \(A\) be a real constant. Consider the quartic polynomial equation

\[x^4 + (4A^2 - A - 100)x^2 + 100A - 4A^3 = 0.\]

The goal of this problem is to determine the number of solutions to the equation by using MATHEMATICA to graphically represent those solutions. This will provide a check to the algebraic work you did in the first team assignment. You can use the MATHEMATICA experiment presented in class as a model for your work.

(a) Create a table of graphs using the MATHEMATICA commands

\[
f[x_] = x^4 + (4*A^2 - A - 100)*x^2 + 100*A - 4*A^3
Table[Plot[f[x], {x,-1,2}], {A,-1,6}]
\]

and then animate the graphs as shown in class. Explain briefly how the command works and why the outcome is not as useful as we might like for analyzing solutions to the equation.

(b) Improve the animation in part (a) by appropriately adding modifiers to the Plot command. (Refer to the MATHEMATICA code of the experiment presented in class but remember that you will probably need to adjust display windows for the different equation we are looking at here.) Include the commands for your animation in the final print-out, but don’t attempt to display any of the graphic output.

(c) Choose at least six different values of \(A\) and display the graphs of \(f\) for those values (do include these pictures in your print-out). Include any special values for \(A\) where the number of solutions changes (that is, where the number of solutions is different for slightly lower and for slightly higher \(A\)-values).

(d) Use the ContourPlot command to sketch a graph of the left-hand side of the equation in the \(xA\)-plane.

(e) Use the work above to give the answer to the question from the first team assignment. Did you come up with the same answer as before? If not, explain.

Problem 2. Let \(A\) be a real constant. Consider the polynomial equation

\[x^N + (4A^2 - A - 100)x^2 + 100A - 4A^3 = 0\]

where \(N\) is a fixed positive integer.

(a) Choose a few different values for \(N\) including \(N = 3, N = 6\) and at least one larger value, and use a MATHEMATICA routine like in part (d) of Problem 1 to give a quick estimate to the number of solutions to the equation for different values of \(A\). Be sure to look at different windows. How can you know that you’ve found a window (or windows) that enable you to be confident that you have correctly determined the number of solutions?

(b) Briefly explain why the algebraic approach you used on the first assignment would be difficult to implement for values of \(N\) larger than 4.