Practice/Review Problems
Calculus 2

Determine whether each of the statements are **True or False**. (Assume throughout that \(f(x)\) and \(g(x)\) are continuous functions). The answers may be found on page 3.

1. If \(c\) is a constant then \(\int cf(x)\,dx = c\int f(x)\,dx\).
2. \(\int_a^b f(x)\,dx = 0\)
3. \(\int_a^b f(x)\,dx - \int_a^c f(x)\,dx = \int_c^b f(x)\,dx\).
4. \(\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx\)
5. \(\int f(x)g(x)\,dx = \int f(x)\,dx \int g(x)\,dx\)
6. \(\int_{-2}^1 \frac{1}{x^4} \,dx = -\frac{3}{8}\)
7. \(\int_0^2 (x - x^3)\,dx\) represents the area between \(y = x - x^3, y = 0, x = 0\) and \(x = 2\).
8. If \(f'\) is continuous on the interval \([1, 3]\) then \(\int_1^3 f'(v)\,dv = f(3) - f(1)\).
9. If \(f(x) = 3\) then each Riemann sum of \(f\) associated with the interval \([0, 2]\) equals 6.
10. If \(f\) is differentiable in \([a, b]\) then \(\int_a^b f(x)\,dx\) exists.
11. \(\int_0^1 (x - 1)^2\,dx = \int_0^1 (t - 1)^2\,dt\)
12. \(\int_0^1 \frac{x^4}{(2 + x^5)^6} \,dx = 7/3\).
13. \(\int (x^2 - 7)^2\,dx = \frac{1}{3}(x^2 - 7)^3 + C\).
14. \(\int_0^2 (x - x^3)\,dx\) represents the area under the curve \(y = x - x^3\) from 0 to 2.
15. The substitution \(u = \sqrt{1 - x}\) changes \(\int_0^1 \sqrt{1 - x}\,dx\) to \(\int_0^2 2u^2\,du\).
16. A rod extends over the interval \([0, 2]\) on the x-axis and has density \(\delta(x) = (x + \sqrt{x})^2\) at \(x\).
   The mass of the rod is \(14/3 + 16\sqrt{2}/5\).
17. \(\int_0^1 (x - 1)^2\,dx = \int_0^1 (x - 1)^2\,dt\)
18. \(\int_1^2 x^2\,dx = \int_1^3 x^2\,dx + \int_3^2 x^2\,dx\)
19. The average value of \(\sqrt{x}\) in \([0, 1]\) is \(2/3\).
20. The average value of \(\sqrt{x}\) in \([0, 4]\) is \(16/3\).
21. If \(c\) is constant then \(\int_a^b c\,dx = c(b - a)\).
22. \(\int_{-4}^4 \sqrt{16 - x^2}\,dx = 8\pi\).
23. The area of the region bounded by $y = 1 - x^2$ and the $x$-axis is $4/3$.

24. Suppose $f$ is an odd function which is continuous on $[-a, a]$ where $a$ is a constant then $\int_{-a}^{a} f(x) \, dx = 0$.

25. Suppose $f$ is any odd function which is continuous on $[-a, a]$ where $a$ is a constant. Then the area between $y = f(x)$, $y = 0$, $-a$ and $a$ is zero.

26. On $[0, 1]$ the average value of the function $f(x) = x^4$ is larger than the average value of the function $g(x) = x^3$.

27. On $[0, 1]$ the average value of the function $f(x) = \sqrt{x}$ is larger than the average value of the function $g(x) = x^3$.

28. The function $f(x) = |x|$ has an antiderivative on the interval $[-1, 1]$.

29. $\int_{-1}^{1} \frac{1}{x} \, dx = -3/2$.

30. If $f(x) = \int_{0}^{x} \sin^2(t) \, dt$ then $f''(x) = \sin(2x)$.

31. If the graph of $f(x)$ passes through the points $(0, 0)$ and $(1, 1)$ and $f'(x)$ is continuous then $\int_{0}^{1} f'(x) \, dx = 1$.

32. $\frac{d}{dx} \left\{ \int_{x}^{1} t^3 \, dt \right\} = -x^3$.

33. $\int_{0}^{\pi/4} \sec^2(t) \, dt = 1$

34. $\int_{1}^{4} x^{-1/2} \, dx = 2$.

35. $\int_{-1}^{1} x^5 - 6x^9 + x^2 \sin(x) \, dx = 0$

36. $\int_{0}^{1} \frac{-2x}{\sqrt{5 - x^2}} \, dx = -\int_{4}^{5} u^{-1/2} \, du$
ANSWERS:
1. True
2. True
3. False, but note that it is true in some cases, for example when $b = c$ or when $f(x) = 0$.
4. True
5. False
6. False
7. False
8. True
9. True
10. True
11. True
12. False
13. False
14. False
15. False
16. True
17. False
18. True
19. True
20. False
21. True
22. True
23. True
24. True
25. False
26. False
27. True
28. True
29. False
30. True
31. True
32. True
33. True
34. True
35. True
36. True