**Problem 1.** Use MATHEMATICA to compute the integral \( \int \sin^3(x) \cos(x) \, dx \). Then compute the integral yourself using a substitution. Which answer do you like better? Find a way to get MATHEMATICA to verify that the two answers are the same.

To get MATHEMATICA to write expressions in different forms you might want to experiment with commands such as Expand, Simplify, FullSimplify, TrigReduce or TrigExpand which you can read about in the Help Master Index.

**Problem 2.** By typing "Integrate[Sqrt[a^2+x^2],x] /. a –> 4" MATHEMATICA will compute the indicated integral and then substitute the value of 4 for the variable \( a \). Compare the output of this command with the output of "Integrate[Sqrt[16+x^2],x]". Explain why the outputs are equal (or get MATHEMATICA to check).

**Problem 3.** (a) Use MATHEMATICA to evaluate the following integrals:

(i) \( \int \frac{1}{(x+2)(x+3)} \, dx \)

(ii) \( \int \frac{1}{(x+1)(x+5)} \, dx \)

(iii) \( \int \frac{1}{(x+2)(x-5)} \, dx \)

(iv) \( \int \frac{1}{(x+2)^2} \, dx \)

(b) Based on the pattern of your responses in part (a) guess the value of \( \int \frac{1}{(x+a)(x+b)} \, dx \) if \( a \neq b \). What if \( a = b \)?

**Problem 4.** (a) Use MATHEMATICA to evaluate the following integrals:

(i) \( \int \ln(x) \, dx \)

(ii) \( \int x \ln(x) \, dx \)

(iii) \( \int x^2 \ln(x) \, dx \)

(iv) \( \int x^3 \ln(x) \, dx \)

(v) \( \int x^7 \ln(x) \, dx \)

(b) Based on your responses to part (a) guess the value of the integral \( \int x^n \ln(x) \, dx \).

**Problem 5 (EXTRA).** Let \( f(x) = x/3+1 \), and for each positive integer \( n \) let \( f_n(x) \) be the function obtained by composing \( f \) with itself \( n \) times. So \( f_2(x) = f(f(x)) = (x/3+1)/3+1 = x/9+4/3 \) and etc. Use MATHEMATICA to determine the value of the definite integral of \( f_n(x) \) on the interval \([0,1]\) for at least ten selected values of \( n \), and use your answer to conjecture what happens as \( n \) limits to \(+\infty\).

There is a command in MATHEMATICA "Nest" which will compose a function with itself any number of times. For example, \( Nest[f, x, 3] \) represents \( f(f(f(x))) \).