Assignment #4  2-22-05

p. 78  S. J. S.
(Due in general matrix for (from (4) on p. 62):

\[ \begin{bmatrix} a & b & c & \ldots & j & k \end{bmatrix} \]

When \((a b c d e f g h i j k)^T\) is multiplied by the general matrix, rows 1 through 11 give the

\[ \begin{align*}
  u &= a + d + e + g + h + i + j + k \\
  x &= a + b + c + d + e + g + h + i \\
  y &= b + c + e + g + h + i + j + k \\
  z &= c + d + e + g + h + i + j + k
\end{align*} \]

In the following table, draw a "check" if the

variable \(a-\) \(k \) appears in \(u-x\):

\[
\begin{array}{cccccccc}
  a & b & c & d & e & f & g & h \\
  u & V & V & V & V & V & V & V \\
  x & V & V & V & V & V & V & V \\
  y & V & V & V & V & V & V & V \\
  z & V & V & V & V & V & V & V \\
\end{array}
\]

Notice that every column of the table has a different pattern of

checks. If the element of the first column is changed, then

the element among \(a x g \) that change are those checked.

Since all columns are different, any single error in \(a-\) \(k \) changes \(a x g \) differently. And any single error among

\(a x g \) changes \(a x g \) differently as well. Thus, any

single error changes a different set of the relations \(\star\),

errors in \(a-\) \(k \) changing at least 2, errors in \(a x g \)

changing only one.
5.3. (Refer to Theorem 2.19.) If \( d(H') = d(K') = 4 \),

then \( H' \) & \( K' \) can correct \( t \) errors and detect \( \leq t + 1 \) errors.

If \( d(H') \geq 2S + t + 1 \), \( d(K') \geq 2S + t + 1 \),

and \( 4 = 2S + 1 + 1 \) (\( \text{i.e., } S = t = 1 \)), \( H' \) will correct \( 5 \) errors and detect \( 2 \). By examining the codewords in page 7,

we see \( d(H') = d(K') = 4 \).

Now suppose \( u \in H' \) and \( \text{wt}(u) = 4 \). Then \( \exists \ v \in \mathbb{Z}_2^8 \)

such that \( d(u, v) = 2 = d(v, 0) \). (\( \text{wt}(v) \)). Then

if \( v \) is received, it differs by 2 errors from 2 codewords,

\( u \) and \( 0 \). So we cannot correct 2 errors made in \( u \), in general.

Thus, reasoning applies to \( K' \).