Homework 2 Solutions

1. $f(-3) = ?$

$-3 < 2$ so

$f(-3) = 3 - \frac{1}{2}(-3)$

$= 3 + \frac{3}{2}$

$= \frac{6+3}{2}$

$= \boxed{\frac{9}{2}}$

$f(0) = ?$

$0 < 2$ so

$f(0) = 3 - \frac{1}{2}(0)$

$= \boxed{3}$

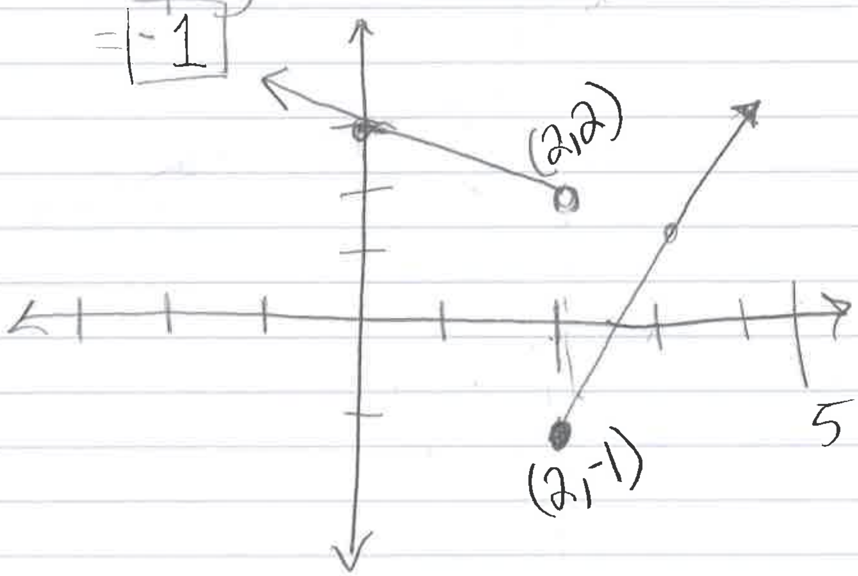
$f(2) = ?$

$2 \geq 2$ so

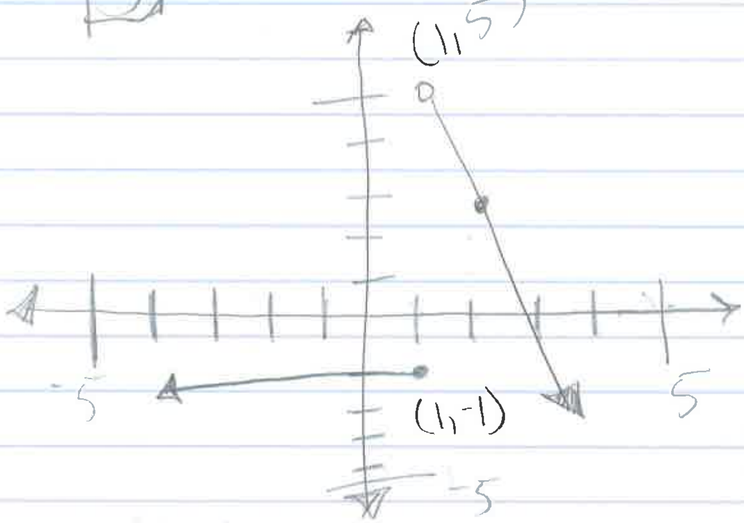
$f(2) = 2(2) - 5$

$= 4 - 5$

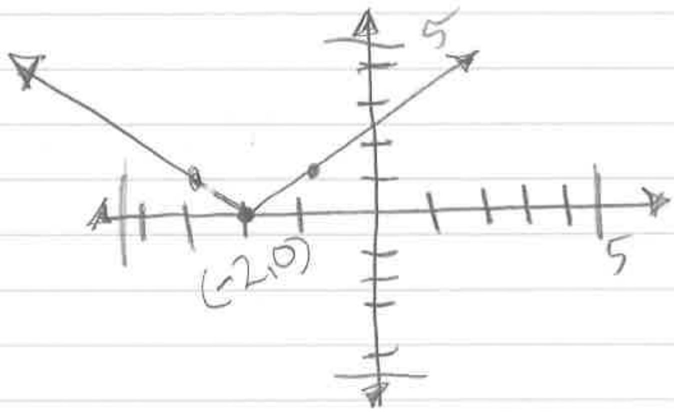
$= \boxed{-1}$



44. $-3 \le 1$ so
$f(-3) = \boxed{-1}$
$0 \le 1$ so
$f(0) = \boxed{1}$
$2 > 1$ so
$f(2) = 7 - 2(2)$
$= 7 - 4$
$= \boxed{3}$



3. $f(x) = |x+2|$
Recall,
$$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$
So,
$$|x+2| = \begin{cases} x+2 & \text{if } x+2 \ge 0 \\ -(x+2) & \text{if } x+2 < 0 \end{cases}$$
Simplifying we get:
$$|x+2| = \begin{cases} x+2 & \text{if } x \ge -2 \\ -x-2 & \text{if } x < -2 \end{cases}$$
p.2 Now we graph,



4. $h(t) = |t| + |t+1|$

Note:

$$|t| = \begin{cases} t & \text{if } t \geq 0 \\ -t & \text{if } t < 0 \end{cases} \quad \&$$

$$|t+1| = \begin{cases} t+1 & \text{if } t+1 \geq 0 \\ -(t+1) & \text{if } t+1 < 0 \end{cases}$$

$$= \begin{cases} t+1 & \text{if } t \geq -1 \\ -t-1 & \text{if } t < -1 \end{cases}$$

So now I want to write $|t| + |t+1|$.

Whatever number I plug in I add these two function values together.

So, if I plug in a number, $t < -1$ we get

$$h(t) = |t| + |t+1|$$
$$= -t + -t-1$$
$$= -2t - 1$$

When I plug in a number, $-1 \leq t < 0$, we get

$$h(t) = |t| + |t+1|$$
$$= -t + t+1$$
$$= 1$$

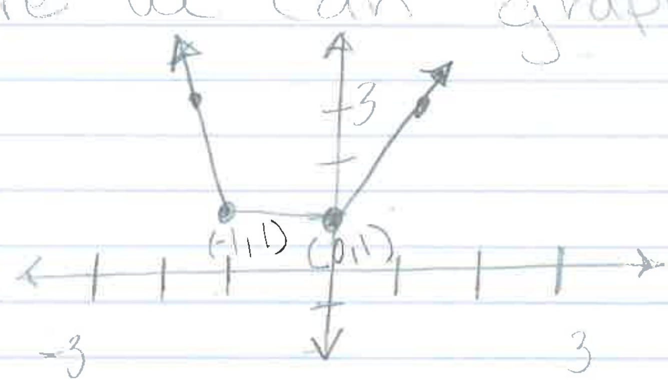
When I plug in a number, $t \geq 0$ we get

$$h(t) = |t| + |t+1| = t + t+1 = 2t+1$$

Putting everything together we get

$$h(t) = \begin{cases} 2t+1 & \text{if } t \geq 0 \\ 1 & \text{if } -1 \leq t < 0 \\ -2t-1 & \text{if } t < -1 \end{cases}$$

Therefore we can graph as



5. First I find the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10-10}{7-(-5)} = \frac{-20}{12} = -\frac{5}{4}$$

$$\boxed{(y-10) = \frac{-5}{4}(x+5)}$$

OR $$\boxed{(y+10) = \frac{-5}{4}(x-7)}$$

OR

$$y = \frac{-5}{4}x - \frac{25}{4} + 10$$

$$y = \frac{-5}{4}x \frac{-25+40}{4}$$

$$\boxed{y = \frac{-5}{4}x + \frac{15}{4}}$$

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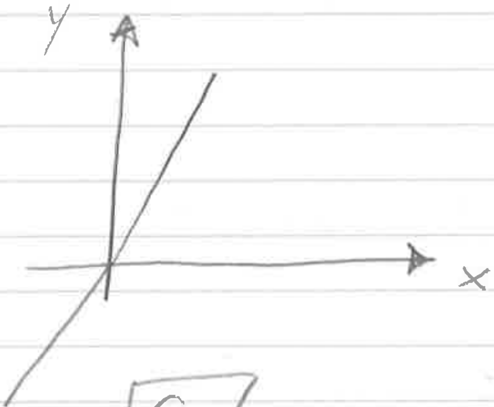
6. (a) $y = 3x$ is a line with slope 3 & y-intercept 0, so it should look like



which is $\boxed{G}$

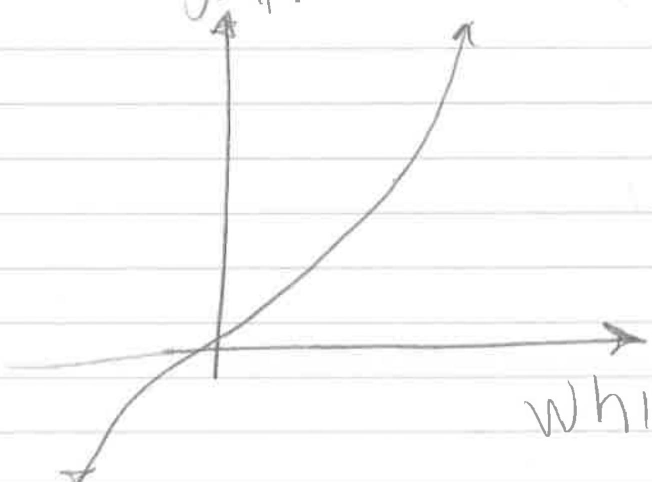
(b) $y = 3^x$

This is exponential, so it should look something like

Which is $\boxed{F}$

(c) $y = x^3$

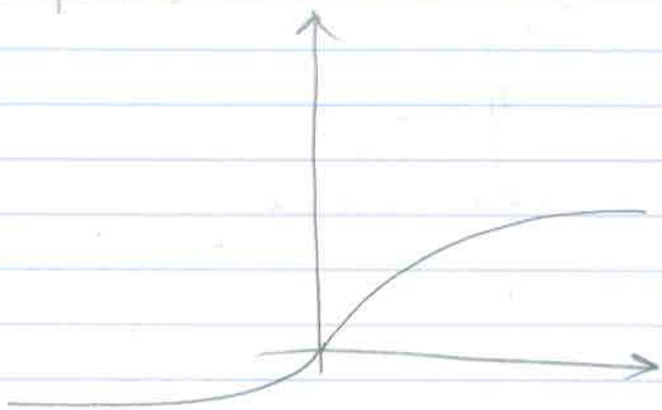
We graphed this in class. It looks like:



which is $\boxed{F}$

(d) $y = \sqrt[3]{x}$

This is a radical, so its graph looks something like



which is $\boxed{g}$

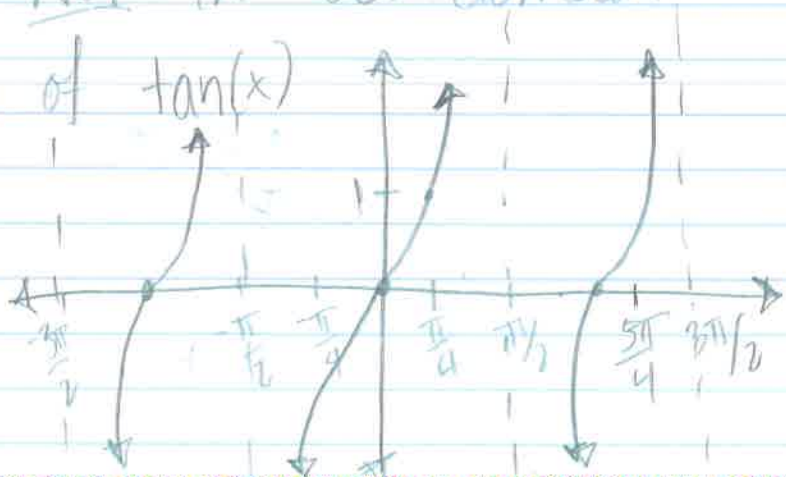
7. $g(x) = \dfrac{1}{1 - \tan(x)}$

To find the domain of a fraction, set the denominator equal to 0:

$$1 - \tan(x) = 0$$
$$-\tan(x) = -1$$
$$\tan(x) = 1$$

So the values where $\tan(x) = 1$ are the values <u>NOT</u> in our domain.

Graph of $\tan(x)$



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I need to find the values where $\tan(x) = 1$

These are the values $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \ldots$

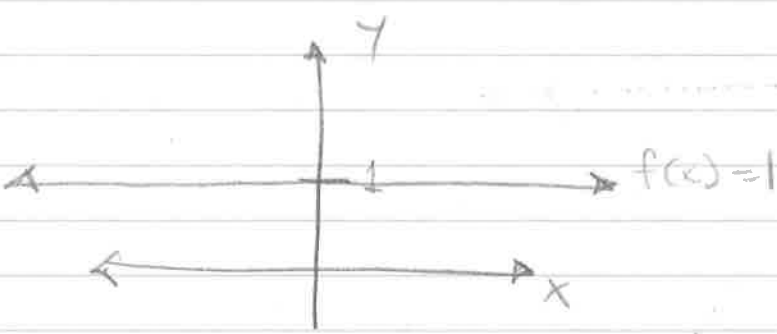
So if $x = \frac{\pi}{4} + k\frac{\pi}{2}$ where k is an integer then $\tan(x) = 1$. So my domain is

$$\boxed{x \neq \frac{\pi}{4} + k\frac{\pi}{2}}$$

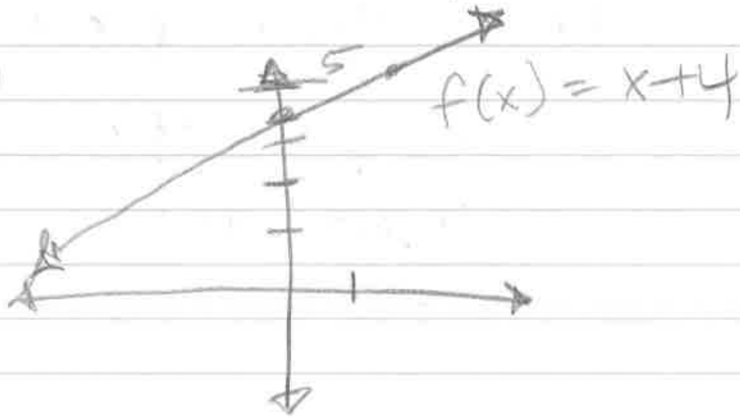
8. $f(x) = 1 + m(x+3)$
$= 1 + mx + 3m$
$= mx + (3m+1)$

This is a $\boxed{\text{straight line}}$ with $\boxed{\text{slope } m}$.

Let $m = 0$
$f(x) = 1$



Let $m = 1$
$f(x) = 1 + (x+3)$
$= 1 + x + 3$
$= x + 4$



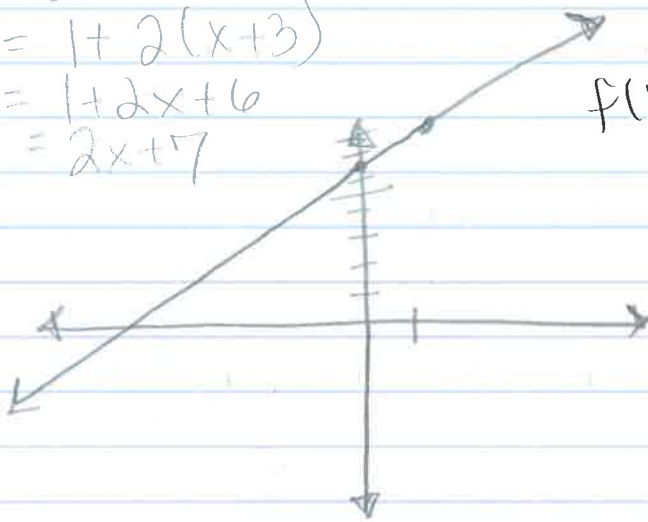
Let $m=2$

$f(x) = 1 + 2(x+3)$

$= 1 + 2x + 6$

$= 2x + 7$

$f(x) = 2x + 7$

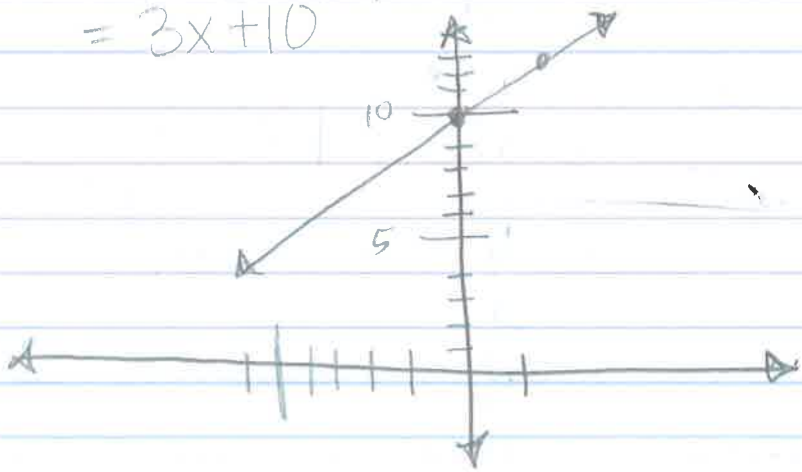


Let $m=3$

$f(x) = 1 + 3(x+3)$

$= 1 + 3x + 9$

$= 3x + 10$



9. Cost \$2200 to make 100 chairs
\$4800 to make 300 chairs

$m = \dfrac{4800 - 2200}{300 - 100}$

$= \dfrac{2600}{200}$

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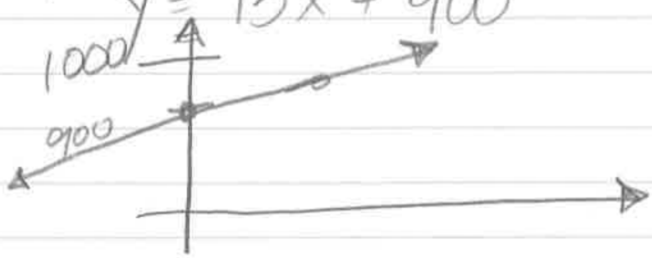
$= 13$
(Assuming my points are (100, 2200) & (300, 4800))

so my function is
$y - 2200 = 13(x - 100)$

$\boxed{y = 13(x-100) + 2200}$

$y = 13x - 1300 + 2200$
$y = 13x + 900$



(b) Slope is <u>13</u> meaning that for every chair you make it costs about \$13 more dollars.

(c) The y-intercept is 900. It represents the cost to make 0 chairs. So if the factory is just running but not producing any chairs.

10. (a) $y = f(x) + 8$ is $f(x)$ shifted <u>up</u> by 8 units

(b) $y = f(x+8)$ is $f(x)$ shifted <u>left</u> by 8 units

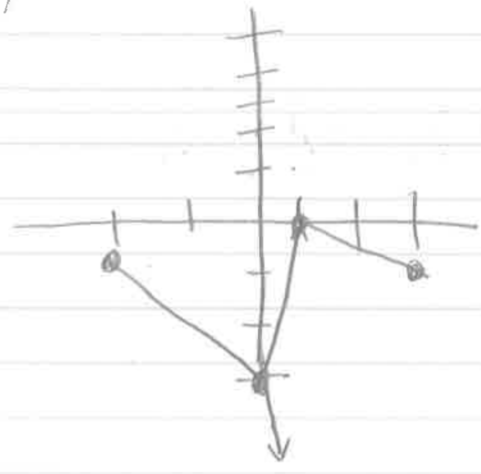
(c) $y = 8f(x)$ is $f(x)$ stretched vertically by 8

(d) $y = f(8x)$ is $f(x)$ shrink horizontally by 8

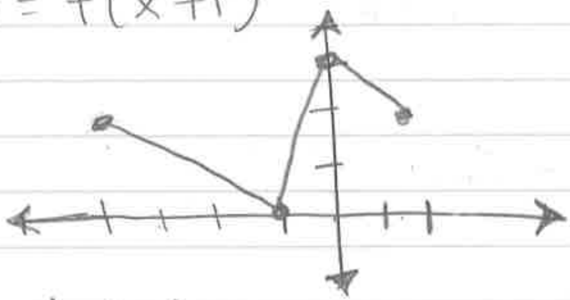
(e) $y = -f(x) - 1$ is $f(x)$ reflected across the x-axis & shifted down by 1.

(f) $y = 8f(1/8\, x)$ is $f(x)$ stretched vertically by 8 & shrink horizontally by 8

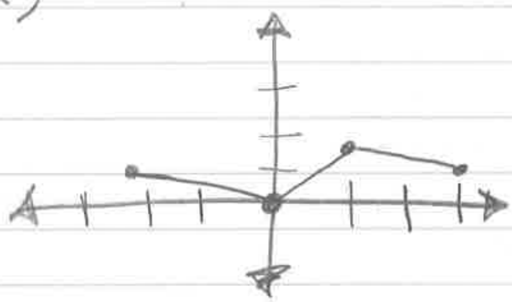
11. (a) $y = f(x) - 3$



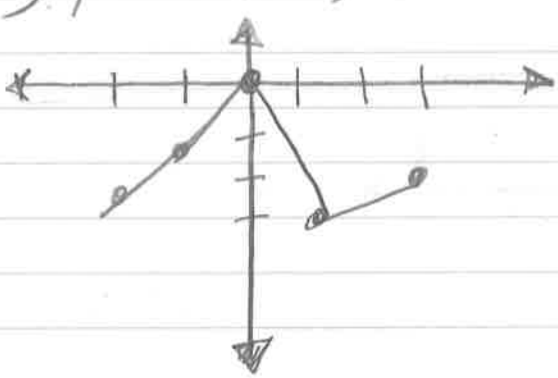
(b) $y = f(x+1)$



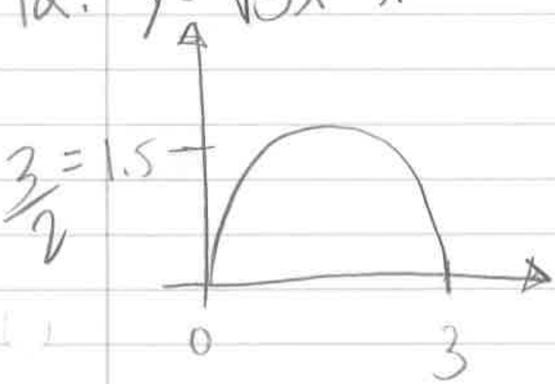
(c) $y = \frac{1}{2}f(x)$



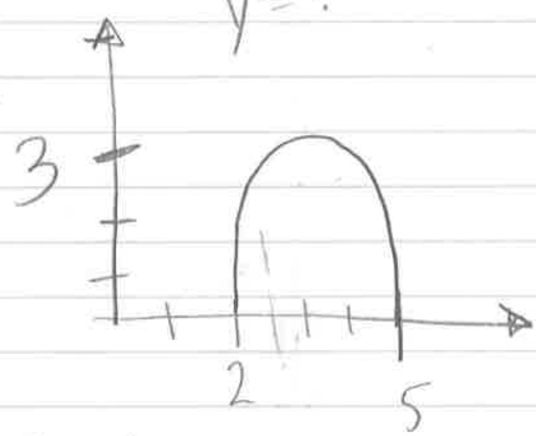
(d) $y = -f(x)$



12. $y = \sqrt{3x - x^2}$



$y = ?$



We have stretched vertically by 2 & moved to the right by 2

So $y = 2\sqrt{3(x-2)-(x-2)^2}$

$= 2\sqrt{3x-6-(x^2-4x+4)}$

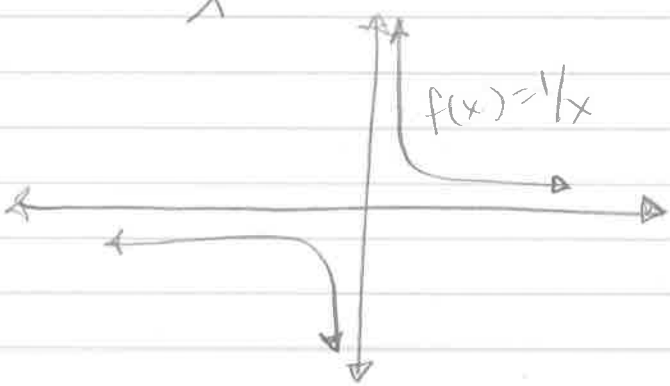
$= 2\sqrt{3x-6-x^2+4x-4}$

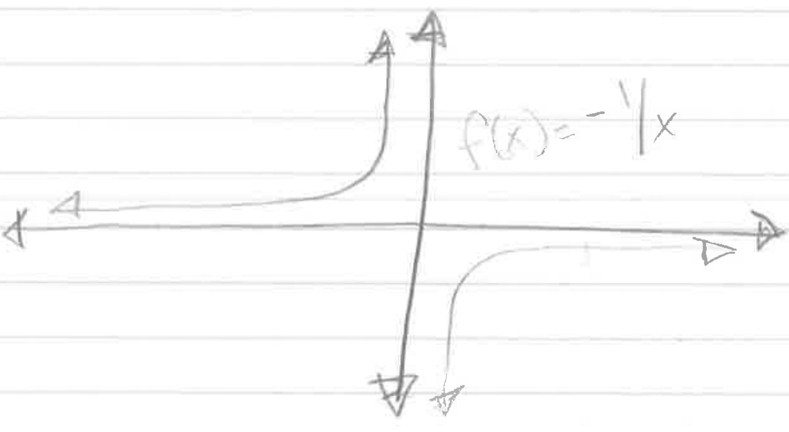
$= 2\sqrt{-x^2+7x-10}$

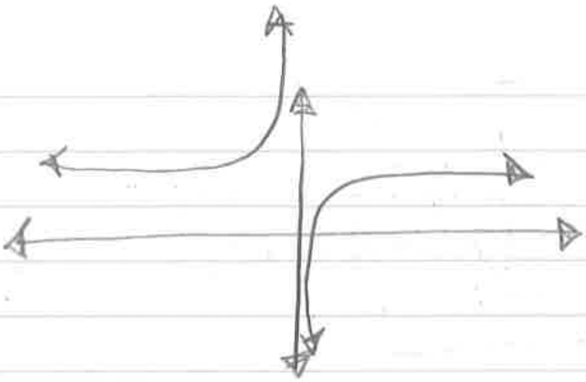
$= 2\sqrt{-(x^2-7x+10)}$

$= \boxed{2\sqrt{-(x-5)(x-2)}}$

13. $y = 1 - \dfrac{1}{x}$

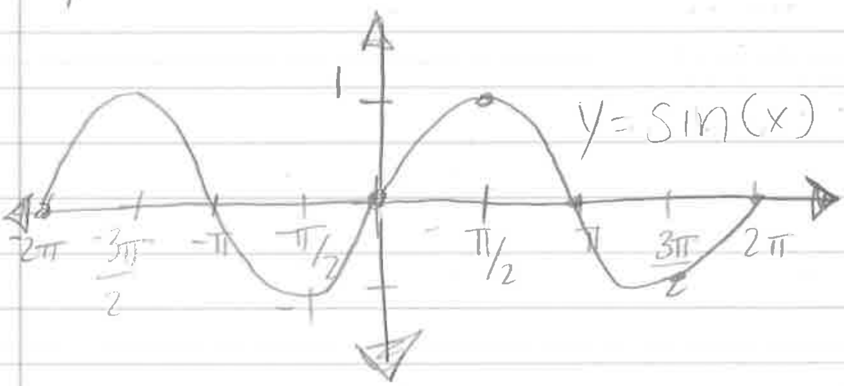




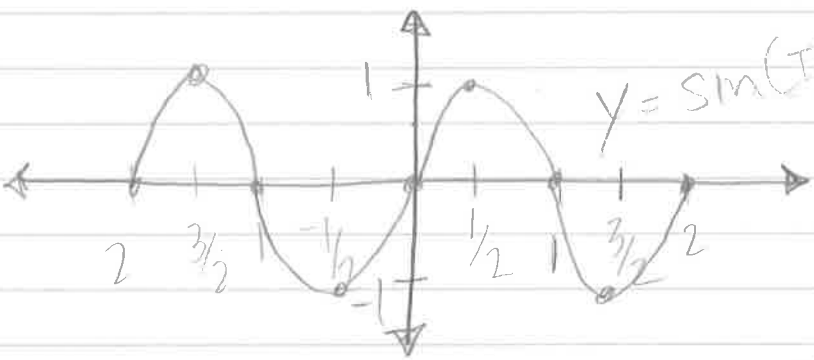


$f(x) = 1 - \frac{1}{x}$

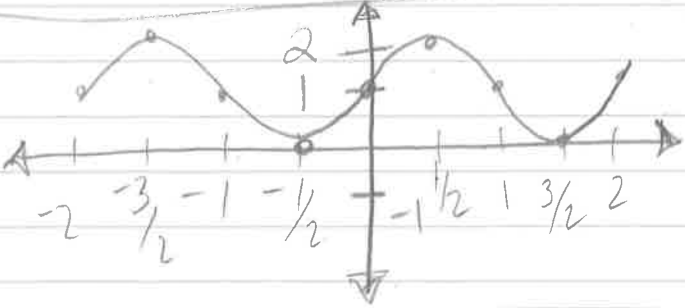
14. $y = 1 + \sin(\pi x)$



$y = \sin(x)$



$y = \sin(\pi x)$



$y = 1 + \sin(\pi x)$

15. (a) $f(g(x)) = \boxed{(1-4x)^3 - 2}$

$= 1 - 3(4)x + 3(16)x^2 - 64x^3 - 2$

$= 1 - 12x + 48x^2 - 64x^3 - 2$

$= \boxed{-1 - 12x + 48x^2 - 64x^3}$

(i) Domain of $g(x)$ $(-\infty, \infty)$

(ii) Domain of $f(x)$: $(-\infty, \infty)$,

Domain of $f(g(x)) = (-\infty, \infty)$

(iii) Domain of $f(g(x))$: $(-\infty, \infty)$

Domain $\boxed{(-\infty, \infty)}$

(b) $g(f(x)) = 1 - 4(x^3 - 2)$

$= 1 - 4x^3 + 8$

$= \boxed{-4x^3 + 9}$

(i) Domain of $f(x)$: $(-\infty, \infty)$

(ii) Domain of $g(x)$: $(-\infty, \infty)$

(iii) Domain of $g \circ f$: $(-\infty, \infty)$

$\boxed{\text{Domain: } (-\infty, \infty)}$

(c) $f \circ f = f(f(x)) = \boxed{(x^3 - 2)^3 - 2}$

$\boxed{\text{Domain: } (-\infty, \infty)}$

(d) $g \circ g = 1 - 4(1 - 4x)$

$= 1 - 4 + 16x$

$= \boxed{-3 + 16x}$

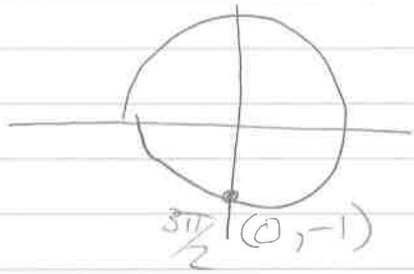
$\boxed{\text{Domain: } (-\infty, \infty)}$

16. (a) $\boxed{f(g(x)) = \dfrac{\sin(2x)}{1+\sin(2x)}}$

~~domain~~: (i) $g(x)$: $(-\infty, \infty)$

(ii) $f(x)$: $x \neq -1 \rightarrow f(g(x))$: $g(x) \neq -1$

For what x is $\sin(2x) = -1$? $\sin(2x) \neq -1$



At $\Theta = 3\pi/2$ $\sin(3\pi/2) = -1$ so when

$2x = \dfrac{3\pi}{2}$

$x = \dfrac{3\pi}{4}$

So $\sin(2x) = -1$ when $X = 3\pi/4$ or any 2π multiple

Thus:

$\boxed{\text{Domain: } x \neq \dfrac{3\pi}{4} + 2\pi k}$

(b) $g(f(x)) = \sin\left(2\left(\dfrac{x}{1+x}\right)\right)$

$= \boxed{\sin\left(\dfrac{2x}{1+x}\right)}$

Domain: (i) $x \neq -1$

(ii) $(-\infty, \infty)$

(iii) $x \neq -1$

So

Domain $\boxed{x \neq -1 \text{ or } (-\infty, -1) \cup (-1, \infty)}$

(c) $f \circ f = \dfrac{\frac{1}{1+x}}{1+\frac{1}{1+x}} = \dfrac{\frac{1}{1+x}}{\frac{1+x+1}{1+x}}$

$= \dfrac{1}{1+x} \div \dfrac{2+x}{1+x}$

$= \dfrac{1}{1+x} \cdot \dfrac{1+x}{2+x}$

$= \boxed{\dfrac{1}{2+x}}$

Domain (i) $f(x)$: $x \neq 1$

(ii) $f(x) \neq 1$ $\quad \dfrac{1}{1+x} \neq 1$

so $x \neq 0$

(iii) $\dfrac{1}{2+x}$, $x \neq -2$

$\boxed{(-\infty,-2)\cup(-2,0)\cup(0,1)\cup(1,\infty)}$

(d) $g \circ g = \boxed{\sin(2\sin(2x))}$

Domain $\boxed{(-\infty,\infty)}$

17. $f \circ g \circ h(x) = f(g(h(x)))$

$= f(2^{\sqrt{x}})$

$= \boxed{|2^{\sqrt{x}}-4|}$

18. $u(t) = \dfrac{\tan t}{1+\tan t}$

Let $\boxed{f(x) = \tan(t) \quad \& \quad g(x) = \dfrac{x}{1+x}}$

19. g is odd $h = f \circ g$

Is h odd?

Ex: Let $f(x) = x^2$ & $g(x) = x^3$ (odd)

Then $h = f \circ g = (x^3)^2 = x^6$

which is even

$\boxed{\text{No}}$

What if f is odd?

odd means $h(-x) = -h(x)$

Both f & g are odd so

$f(-x) = -f(x)$

$g(-x) = -g(x)$

$h(-x) = f(g(-x)) = f(-g(x)) = -f(g(x)) = -h(x)$

so $\boxed{\text{h is odd.}}$

What if f is even?

Then $f(-x) = f(x)$

$h(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = h(x)$

so $\boxed{\text{h is even}}$

20. velocity: 10 m/s height: $y = 10t - 1.86t^2$

(a) Average velocity: $\frac{y(b) - y(a)}{b - a}$

(i) [1,2]: $A.V. = \frac{12.56 - 8.14}{1}$

$= \boxed{4.42}$

(ii) [1, 1.5] $A.V. = \frac{10.815 - 8.14}{0.5}$

$= \boxed{5.35}$

(iii) [1, 1.1] $A.V = \frac{8.7494 - 8.14}{0.1}$

$= \boxed{6.094}$

(iv) [1, 1.01] $A.V = \frac{8.202614 - 8.14}{0.01}$

$= \boxed{6.2614}$

(v) [1, 1.001] $= \frac{8.14627814 - 8.14}{.001}$

$= \boxed{6.27814}$

(b) $\boxed{6.3 \text{ m/s}}$

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21. $\lim_{x \to 1^-} f(x) = 3$ — As x nears 1 from the left, f(x) nears 3

$\lim_{x \to 1^+} f(x) = 7$ — As x nears 1 from the right, f(x) nears 7

$\boxed{\text{No}}$, $\lim_{x \to 1} f(x)$ DNE

22. (a) $\lim_{x \to 2^-} f(x) = \boxed{3}$

(b) $\boxed{1}$

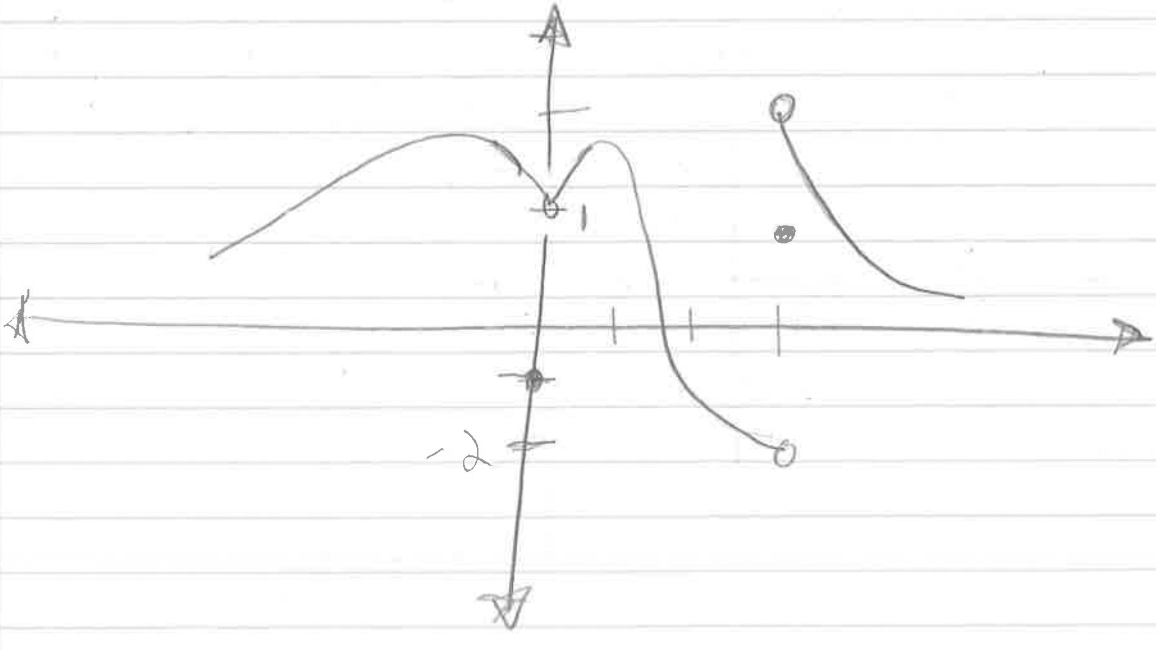
(c) $\boxed{\text{DNE}}$ (because (a) & (b) are different)

(d) $\boxed{3}$

(e) $\boxed{4}$

(f) $\boxed{\text{DNE}}$

23.



24. $\lim\limits_{x \to 5^-} \frac{x+1}{x-5}$

- denominator is getting smaller, so fraction getting larger & negative so

$\lim\limits_{x \to 5^-} \frac{x+1}{x-5} = \boxed{-\infty}$

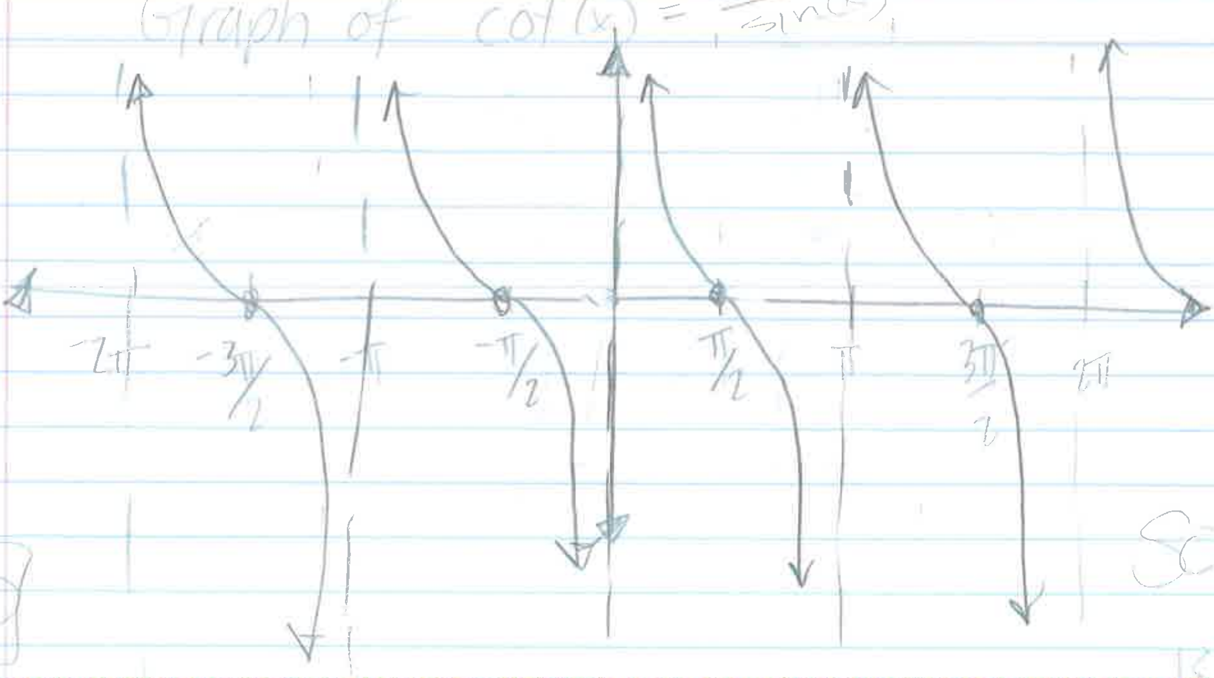
25. $\lim\limits_{x \to 0} \frac{x-1}{x^2(x+2)}$

denominator getting smaller, so whole fraction getting larger & positive

$\lim\limits_{x \to 0} \frac{x-1}{x^2(x+2)} = \boxed{\infty}$

26. $\lim\limits_{x \to \pi^-} \cot(x) =$

Graph of $\cot(x) = \frac{\cos(x)}{\sin(x)}$



So $\lim\limits_{x \to \pi^-} \cot(x) = \boxed{-\infty}$