Number Theory Fall 2009
Homework 7
Due: Wed. Oct. 21, start of class

6.2 Divisibility and primes in $\mathbb{Z}[i]$ and $\mathbb{Z}$

Exercise 6.5. *Factor* 17 *and* 53 *in* $\mathbb{Z}[i]$. *(Exercise 6.2.4.)*

6.3 Conjugates

Exercise 6.6. *Suppose* $p \in \mathbb{N}$ *is prime in* $\mathbb{Z}$ *but factors as* $p = \alpha\beta$ *where* α, β *are non-units in* $\mathbb{Z}[i]$. *Show* $\beta = \overline{\alpha}$.

6.4 Division in $\mathbb{Z}[i]$

Exercise 6.7. *Use the Euclidean algorithm to determine a gcd for* $\alpha = 5$ *and* $\beta = 3 + 4i$ *in* $\mathbb{Z}[i]$.

Exercise 6.8. *Let* $\pi, \beta \in \mathbb{Z}[i]$ *and* u *be a unit of* $\mathbb{Z}[i]$. *Show that* $\pi | u\beta \iff \pi | \beta$.

Exercise 6.9. *Suppose* π *and* π' *are primes of* $\mathbb{Z}[i]$. *Show* $\pi | \pi'$ *implies* $\pi = u\pi'$ *where* u *is a unit of* $\mathbb{Z}[i]$.

Exercise 6.10. *Let* u *be a unit in* $\mathbb{Z}[i]$. *Show* π *is prime in* $\mathbb{Z}[i]$ *if and only if* $u\pi$ *is prime in* $\mathbb{Z}[i]$.

Exercise 6.11. *Suppose* α *is prime in* $\mathbb{Z}[i]$. *Show* $N(\alpha) = p$ *or* $N(\alpha) = p^2$ *for some prime* p *of* $\mathbb{N}$.