

Elliptic Curves: Problem Set 2 (due Fri Mar 10)

Notation: k denotes a perfect field.

Topics: Basics of elliptic curves (Sections 1–5 in Chapter II of Milne)

1. Let C_n be the curve over k in \mathbb{P}^2 defined by $X^3 + Y^3 = nZ^3$.
 - (a) When does C_n define an elliptic curve?
 - (b) When it does, give a Weierstrass equation for C_n .
2. Let $c \in \mathbb{F}_p^\times$, and C/\mathbb{F}_p be the curve in \mathbb{P}^2 defined by $Y^2Z = X^3 + cX^2Z$. Show the number of nonsingular points in $C(\mathbb{F}_p)$ is $p - 1$ or $p + 1$, according to whether c is a square in \mathbb{F}_p^\times or not.

(Note that this proves the stated criteria for split/nonsplit multiplicative reduction given the classification of smooth affine curves with a group structure.)
3. Exercise II.3.2 from Milne
4. Exercise II.3.3 from Milne
5. Exercise II.5.12 from Milne, and check your answers in Sage