Review for Exam 3

Math 2423

April 18, 2016

- 6.6: You are expected to know how to derive the derivatives for $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ when given the general inverse derivative formula $\frac{\mathrm{d}}{\mathrm{d}x}^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$. Eventually you should be able to use these derivatives to evaluate integrals, but these formulas are hard to remember and even harder to identify. The only thing expected of you is to be able to identify what kind of trig substitution you need to make, as mentioned later in Sec 7.3, and then you should be able to reach an answer that is exactly same as the formula without having to memorize them.

- 6.8: The limit of a quotient, once it's in the form of $\frac{\infty}{\infty}$ or $\frac{0}{0}$, can be evaluated by using L'Hospital's Rule. Even when you don't have a quotient, you can turn products into quotients as long as it will be in either form above. The limit of an exponential term can sometimes be found by applying the natural logarithmic function first and then using the L'Hospital's Rule.

- 7.1: We introduced integration by parts in this section. This technique is very useful in several situations: 1) When the integrand has only one factor, we can write it as a product of itself with 1, and apply integration by parts. 2) When the integrand is the product of two functions, one of which is a polynomial, then by successively applying integration by parts one can reduce the degree of the polynomial. 3) The interesting case is when you need to apply integration by parts more than once to get an equation of the original integral, and solve for the integral.

- 7.2: For trig integration, we mainly focused on the two possible cases: $\int \sin^n x \cos^m x \mathrm{d}x$ and $\int \tan^n x \sec^m x \, \mathrm{d}x$. The techniques depend heavily on the parity (either even or odd) of the powers, and either 1) you are lucky, in which case you can use a simple u-sub or 2) you are less lucky, in which you need to use a trigonometric identity to rewrite the function or 3) you can use integration by parts to write an equation of the original integral. In case 2), the double angle formula can be used as long as you can recall them during the test. For 3), the solution you get might have other trig integrals you need to evaluate again, and it should be a much easier problem than the original one.

- 7.3: We introduced three different cases that invoke trig substitutions. Please refer to the textbook or notes for the table. It might be helpful to try to figure out what substitution to use without looking up the table, because they have strong resemblance to the two trig identities $\sin^2 x + \cos^2 x = 1$ and $\tan^2 x + 1 = \sec^2 x$, which we have used many times in class and in HW. Sometimes you need to factor out a constant to make the constant term equal to 1. In the case when the denominator is a quadratic polynomial which you can't factor, you should be able to write it in terms of $(x+a)^2+b^2$ by completing the squares, and then find the corresponding trig substitution.

- 7.4: Partial fraction decomposition is helpful when we want to take the integral of a rational function. In particular, when the degree of the polynomial in the numerator is less than that in the denominator, the general formula for the decomposition is listed both in the notes and textbook and you are expected to learn them by heart. After clearing the denominators, there is a way of determining the coefficients in the formula by solving a system of linear equations. You are allowed to use the Heaviside method (plugging in certain values of x to both sides of the equation,) however this method doesn't save much time when the denominator has a quadratic factor or repeated factors.

- About formulas: aside from the basic trig identities, the ones that are slightly harder to remember are probably 1) the double angle formula and 2) trig substitution table. For 1), even though the formulas will be given for $\sin^2 x$ and $\cos^2 x$, you should always be able to finish the exam without using the double angle formula; For 2), as mentioned above, if you don't want to memorize, the signs in the trig identities should give you a big hint on what substitution to use. You won't be expected to know the trig identities and derivatives/integrals involving $\csc x$ and $\cot x$, but they are not so hard to remember once you know the formulas for tangent and secant.